

Gauge mechanism of mediation of supersymmetry breaking

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Most phenomenologically acceptable supersymmetric models imply breaking of supersymmetry in a separate sector of fields introduced just for this purpose. Supersymmetry breaking is transferred to the Standard Model due to a certain interaction with this additional sector. In this review we discuss one of the popular mechanisms of such mediation with the key role played by the Standard Model gauge interactions. We consider general principles of gauge mediation and give the critical analysis of specific models together with their phenomenological and cosmological implications. We present also basic facts concerning mechanisms of dynamical supersymmetry breaking.

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1 Introduction

In spite of the fact that no experimental evidence for supersymmetry has been discovered so far, the possibility that nature is supersymmetric attracts continuous attention of scientists working in particle theory. Search for supersymmetry is one of the principal tasks for huge accelerators under construction, particularly, for the Large Hadronic Collider (see, for instance, Ref. [1]). In this review we will discuss one of the approaches to construct realistic supersymmetric theories, which underwent intense exploration in the last few years. It might be useful, however, to recall first why the models that yet have no experimental evidence for, attract so great attention.

Supersymmetry [2] is a symmetry between bosons and fermions which provides a nontrivial extension of Poincare algebra. Supersymmetric quantum field models have been constructed [3, 4] and further investigated in great detail (see, for instance, textbooks Ref. [5, 6] and reviews Ref. [7, 8]). These models turned out to possess a number of features attractive from the particle physics point of view. The main point is that the ultraviolet behaviour of these theories is better than in non-supersymmetric models, that is, additional cancellation of ultraviolet divergencies occurs due to supersymmetry. In particular, quadratically divergent radiative corrections to the masses of scalar particles (e.g., Higgs boson) are absent in these models. This fact is the main motivation for constructing realistic supersymmetric theories. This property allows one to solve the so-called gauge hierarchy problem, which is often considered as the main theoretical drawback of the Standard Model of particle interactions. Namely, suppose that the Standard Model provides a good description of subnuclear physics either up to the energies where quantum gravitational effects become essential (Planck mass, $M_{Pl} \sim 10^{19}$ GeV), or up to the Grand Unification scale ($M_{GUT} \sim 10^{16}$ GeV). Then the characteristic scale of the electroweak symmetry breaking determined by Z or Higgs boson masses $\sim 10^2$ GeV $\ll M_{GUT}, M_{Pl}$ gets huge radiative corrections due to quadratically divergent diagrams. That is, the hierarchy of mass scales is unstable with respect to quantum effects. On the other hand, scalar masses in realistic supersymmetric models get only logarithmic radiative corrections, so the hierarchy of mass scales is stable. The latter fact provides serious support to supersymmetric generalizations of the Standard Model.

Supersymmetry, however, is very restrictive, as far as the particle spectrum is concerned. Namely, together with known particles, their superpartners should exist which are the particles of a different spin but with the same quantum numbers and masses. Since scalar particles with masses of quarks and leptons as well as fermions with masses of gauge bosons are absent, unbroken supersymmetry in nature is ruled out. Nevertheless it is possible to break supersymmetry without spoiling attractive cancellation of quadratic divergencies [9]. This kind of breaking is called soft breaking and yields heavy superpartners in a natural way. These arguments show that broken supersymmetry might be relevant to nature, thus posing the problem of how it is broken.

Softly broken supersymmetry, provided its breaking is not very strong, has one additional advantage — it allows to explain not only stability but even the origin of the gauge hierarchy [10]. Namely, supersymmetry breaking occurs in many models dynamically, due to nonperturbative effects. The latter are important at scales $\Lambda \sim \exp(-O(1/g^2)) M$, where $M \sim M_{Pl}$ or M_{GUT} , and g is some coupling constant. If g is small enough, then the supersymmetry breaking scale $\Lambda \ll M$. On the other hand, in most realistic models the vacuum expectation value of Higgs boson is generated by radiative corrections, so it is determined by Λ up to powers of coupling constants. This could provide the origin of gauge hierarchy.

Among the reasons to be interested in supersymmetric particle theories, much better (as compared to the Standard Model) unification of gauge couplings at the scale M_{GUT} is worth mentioning. One should remember also about purely aesthetic advantages of the theories possessing larger symmetries. In practice, this leads to the possibility to gain nontrivial dynamical information by making use of symmetry arguments (see section 5.1).

In the simplest models of supersymmetry breaking, the latter occurs spontaneously at the tree level (O’Raifeartaigh [11] and Fayet – Iliopoulos [12] models). Explaining the origin of gauge hierarchy in these models requires fine tuning of mass parameters, that is, introducing a new hierarchy. Moreover, there is a more fundamental reason to exclude the possibility of direct implementation of these mechanisms in supersymmetric generalizations of the Standard Model. Namely, even in theories with broken supersymmetry, particle spectrum cannot be arbitrary. In the physically interesting case, the so-called supertrace of the mass squared matrix (sum of the masses squared for all particles in a given supermultiplet weighted by plus sign for bosons and minus sign for fermions) is equal to zero, $\text{STr } m^2 = 0$, at tree level (neglecting effects of supergravity). This means that the Standard Model fermions should have scalar superpartners not heavier than fermions themselves, the result being clearly phenomenologically unacceptable. So, realistic theories should overcome one of the conditions of this theorem, that is either to deal with supergravity, or to exploit loop effects in renormalizable models. The attempts to use the second approach yielded the gauge mediation mechanism, the subject of this review.

General problems of supersymmetric particle theories will not be considered here in detail since they are carefully discussed, for instance, in the reviews [8, 13, 14]. Dynamical supersymmetry breaking is considered also in the reviews [15, 16], while gauge mediation is discussed in Refs. [17, 18].

In section 2, basic notations are introduced and the minimal supersymmetric extension of the Standard Model (MSSM) is discussed in brief. In section 3, we discuss the mechanism of gauge mediation of supersymmetry breaking and phenomenology of the simplest, Minimal model, which exhibits most characteristic features of gauge mediation models. Extensions of the Minimal model and specific aspects of their phenomenology are discussed in section 4. Section 5 is devoted to different realizations of an attractive feature of gauge mediation models — the possibility to construct and

analyse quantitatively a full theory incorporating both dynamical supersymmetry breaking and its mediation to MSSM fields. Section 6 deals with cosmological implications, generic to models under discussion, and with constraints on mechanisms of supersymmetry breaking and its mediation coming from cosmology. In Conclusion, we outline main features of gauge mediated supersymmetry breaking models and mention a few problems interesting for future investigation.

2 Basics of the MSSM

Minimal supersymmetric standard model (MSSM) is usually considered as a low energy theory describing phenomenology of supersymmetric models. Standard formalism of supersymmetric models of particle physics is discussed in detail in reviews and textbooks, Refs. [5, 6, 7, 8], while various aspects of MSSM are considered, for instance, in reviews [8, 19, 20]. In this section, we will briefly discuss the basic features of the MSSM and introduce our notations.

MSSM Lagrangian is naturally separated in two parts, the first being supersymmetric and the second containing soft supersymmetry breaking terms. Let us consider the supersymmetric part first. A convenient formalism to deal with supersymmetric Lagrangians is the superspace technique. In this formalism, functions depend not only on usual space-time coordinates x^μ but also on anticommuting coordinates θ_α and their conjugates, $\bar{\theta}_{\dot{\alpha}}$, which are two-component spinors of the Poincare algebra. These functions are called superfields; usual fields appear as coefficients of expansion of superfields in series in θ_α , $\bar{\theta}_{\dot{\alpha}}$. Since θ_α , $\bar{\theta}_{\dot{\alpha}}$ are anticommuting, these series are in fact finite polynomials. Action of the fields is an ordinary integral of the Lagrangian density with respect to space-time coordinates and Berezin integral [21] with respect to supercoordinates.

To construct a realistic supersymmetric model, it is necessary to ascribe a superfield to each field of the Standard Model, the fact which leads to the presence of new particles in the spectrum. These new particles (superpartners) carry the same global quantum numbers as the ordinary fields but different spin.

Gauge fields¹ are described by vector (real) superfields. In the most commonly used Wess-Zumino gauge, a real superfield \mathcal{V} consists of a usual vector field A_μ , its superpartner (gaugino) — fermion λ in the adjoint representation of the gauge group — and an auxiliary scalar field D , the latter having no kinetic term so it may be integrated out by means of equations of motion. In superfield formulation, the action reads

$$S_g = \text{Im} \left[\tau \int d^2\theta d^4x \text{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) \right], \quad (1)$$

where \mathcal{W}_α is a superfield containing the gauge stress tensor $F_{\mu\nu}$ and $\tau = \frac{\Theta}{2\pi} + i\frac{4\pi}{g^2}$ is a combination of the gauge coupling g and the CP -violating parameter Θ . Written

¹Hereafter we consider only the simplest, $N = 1$ supersymmetry.

in terms of components, this is the usual gauge-invariant action of gauge bosons and massless gauginos with Θ -term included. In the MSSM, the names of superpartners of $SU(3) \times SU(2) \times U(1)_Y$ gauge bosons are gluino (for $SU(3)$ gluons), wino and zino (for weak W and Z bosons) and bino (for $U(1)_Y$). Photon's superpartner is photino (for the electromagnetic gauge group $U(1)_{\text{EM}}$ which remains unbroken after the electroweak symmetry breaking).

Matter fields are described by another representation of the superalgebra, the chiral superfields

$$\Phi = \phi(y) + \sqrt{2}\psi(y)\theta + \theta\theta F(y) ,$$

where ϕ is a scalar component of the superfield, ψ is a fermion and F is an auxiliary scalar field. Supersymmetric action which describes interactions of chiral superfields (with each other and with gauge superfields) is given by the functional,

$$S_m = \int d^2\bar{\theta}d^2\theta d^4x \Phi^\dagger e^\gamma \Phi + \left(\int d^2\theta W(\Phi) + h.c. \right) . \quad (2)$$

The first term is called Kahler potential; it contains gauge invariant kinetic terms for ϕ and ψ and their Yukawa interaction with gauginos. The second term in Eq. (2) is called superpotential and contains self-interaction of components of the chiral supermultiplet. The full action is a sum of S_g and S_m . With the auxiliary fields integrated out, scalar potential takes the form

$$V = \sum_i |F_i|^2 + \sum_a \frac{1}{2} D_a^2 , \quad (3)$$

where F -terms are determined from the superpotential:

$$F_i = \frac{\partial W(\phi)}{\partial \phi_i} ,$$

and D -terms in the Wess-Zumino gauge are

$$D^a = \sum_i \phi_i^\dagger T^a \phi_i .$$

(T^a are the gauge group generators).

Quarks and leptons in the MSSM are fermionic components of chiral superfields (their scalar superpartners are called squarks and sleptons) while Higgs fields are the bosonic components of other chiral superfields (their fermionic superpartners are called higgsinos). Note that, contrary to the Standard Model, two Higgs fields are required in the MSSM. Electrically neutral fermions (photino, zino, and neutral higgsino) may mix with each other, forming physically observable linear combinations, neutralinos. Charged fermions, that is winos and charged higgsinos, may mix also.

Superpotential of the MSSM contains terms describing Yukawa interaction of the matter and Higgs fields,

$$W_{MSSM} = H_D(L_i Y_{ij}^l E_j + Q_i Y_{ij}^d D_j) + H_U Q_i Y_{ij}^u U_j + \mu H_U H_D, \quad (4)$$

where L_i , E_i , Q_i , D_i , and U_i are superfields containing left- and right-handed leptons, left- and right-handed down and up quarks, respectively, together with their superpartners; H_D and H_U contain Higgs doublets (hereafter we define superfields by the upper-case, and their scalar components by the lower-case letters); Y_{ij}^l , Y_{ij}^d , Y_{ij}^u are Yukawa mixing matrices. Superpotential (4) respects all symmetries of the MSSM, including lepton and baryon number conservation. Note that it contains the so-called μ -term, a supersymmetric mixing mass term for the Higgs fields.

Generally, additional terms in the superpotential are allowed by $SU(3) \times SU(2) \times U(1)_Y$ gauge symmetry. These terms lead to violation of lepton and baryon numbers. To satisfy experimental constraints coming, for instance, from proton decay, coefficients of these terms should be very small. The usual assumption is that these terms are forbidden by an additional $U(1)_R$ global symmetry. The symmetry is broken down to a discrete Z_2 symmetry called R -parity when supersymmetry is broken. All Standard Model particles have charge $+1$ under this symmetry, while their superpartners carry charge -1 . R parity conservation leads to stability of the lightest superpartner (LSP) of the Standard Model particles.

Besides the MSSM fields, an important role in the phenomenology of gauge mediated supersymmetry breaking is played by the superpartner of graviton — gravitino, a particle of spin $3/2$ which is massless in the limit of unbroken supersymmetry.

Soft supersymmetry breaking terms are added to the MSSM Lagrangian explicitly. These terms contain mass terms for gauginos, $(\frac{1}{2} M_{\lambda_i} \lambda_i \lambda_i + \text{h.c.})$, and for scalar fields, Higgses ² for example,

$$m_{h_U}^2 |h_U|^2 + m_{h_D}^2 |h_D|^2 + (B_\mu h_U h_D + \text{h.c.}), \quad (5)$$

as well as trilinear interactions of the scalar fields,

$$h_D(l_i y_{ij}^l e_j + q_i y_{ij}^d d_j) + h_U q_i y_{ij}^u u_j + \text{h.c.}, \quad (6)$$

where y_{ij} are new matrices of coupling constants. So, about a hundred new parameters are introduced in addition to those of the Standard Model.

Generally, such amount of parameters might lead to nontrivial flavour physics. Absence of flavour-changing neutral currents (FCNC), processes with CP- and lepton flavour violation poses severe constraints on these parameters.

²It can be shown that a strong constraint on the mass of the lightest Higgs boson arises in the MSSM. At tree level, it implies that $m_h < M_Z$. Though it is somewhat relaxed by loop corrections, most realistic models predict the lightest Higgs boson mass not larger than 150 GeV [22]. This distinction from the Standard Model allows the MSSM to be ruled out if the Higgs particle is not discovered at future accelerators.

Usually, these constraints come from consideration of the first two generations. A characteristic example is provided by the system of neutral kaons where absence of FCNC implies [23]

$$\varkappa \left(\frac{1 \text{ TeV}^2}{m_{\tilde{Q}}^2} \right) \left(\frac{\delta m_{\tilde{Q}}^2}{m_{\tilde{Q}}^2} \right)^2 \lesssim 5 \times 10^{-3}, \quad (7)$$

where $\delta m_{\tilde{Q}}$ is a mass difference between u and s squarks, $m_{\tilde{Q}}$ denotes their average mass squared, and \varkappa is a product of elements of the rotation matrix between mass eigenbases of quarks and squarks. Similar bounds appear from other flavour violating processes.

One can see from Eq. (7) that there are three possibilities to satisfy experimental bounds on FCNC contribution. Namely, one may:

1. Consider models with heavy squarks, $m_{\tilde{Q}} \gtrsim 10 \text{ TeV}$.
2. Enforce \varkappa to be small by requiring that mass eigenbases for scalar and fermions are parallel.
3. Provide squark mass degeneracy, $\delta m_{\tilde{Q}}^2 \ll m_{\tilde{Q}}^2$ (this approach is most commonly used).

To choose one of the three possibilities in a natural way, one should understand the origin of the soft terms.

An observation important for model building is that supersymmetry cannot be broken in a phenomenologically acceptable way in a framework of a theory which contains only MSSM fields. Indeed, breaking at the tree level in the MSSM is excluded because of the mentioned above property $\text{STr } m^2 = 0$, while dynamical breaking by means of nonperturbative effects is possible only at the energy scale where QCD is strongly coupled. This scale, however, is a few orders of magnitude less than one required to explain the gauge hierarchy and absence of observed superpartners. So, supersymmetry breaking should occur in a new sector which is introduced exclusively for this purpose and contains either O’Raifeartaigh model or a gauge theory where supersymmetry is broken due to Fayet-Iliopoulos mechanism or dynamically. As it has already been pointed out, to explain not only stability but also the origin of the gauge hierarchy, dynamical supersymmetry breaking is preferred. So, it is usually assumed that there exists some gauge group in addition to the Standard Model. There are some matter fields carrying quantum numbers of this group and dynamics of this new sector provides supersymmetry breaking at the required energy scale. Since no evidence for this sector has been found in the experiments, it is natural to suppose that interaction of MSSM fields with that sector is rather weak and becomes essential at energies not available to current experiments. It is this interaction which is responsible for generating the soft terms. Depending on the nature of this interaction, one distinguishes gravitational mediation at scales of order the Planck scale and gauge

mediation. Effects of supersymmetry breaking are transferred to the MSSM at much lower scales in the latter case, so entire model may be described in terms of field theory. This possibility is an advantage of the gauge mediation mechanism since no reliable theory of Planck scale physics is known. In the low energy mediation models, the supersymmetry breaking sector is often called [24] a *secluded* sector — contrary to a *hidden* sector of gravitational models.

The vacuum energy is zero in the case of unbroken supersymmetry, as a consequence of the supersymmetry algebra. So, Eq. (3) relates supersymmetry breaking to appearance of a nonzero vacuum expectation value of some F - or D - component. In realistic models, supersymmetry is often broken with a nonzero vacuum expectation value F_{DSB} of an auxiliary component of some field in the additional sector. In the gravity mediation case, the corresponding soft masses are of order $\frac{F_{DSB}}{M_{Pl}}$. To make these terms of order the electroweak scale, one needs $F_{DSB} \sim M_{Pl} m_Z$.

Difficulties with flavour violation are usually overcome in the supergravity approach by conjecturing the universality of the soft terms [25]. This conjecture states that, at the gravitational scale, all scalar superpartners of the Standard Model fermions get equal masses m_0 while all gauginos get equal masses $m_{1/2}$. The additional trilinear scalar interaction, Eq. (6), is assumed to be equal, up to some coefficient A_0 with dimensionality of a mass, to Yukawa part of the superpotential,

$$y_{i,j}^{l,d,u} = A_0 Y_{i,j}^{l,d,u}.$$

So, at high energies new sources of flavour violation additional to the Standard Model do not appear — scalar masses are degenerate, and GIM mechanism is operating. Nevertheless, in the low energy theory flavour violation occurs not only due to Cabibbo-Kobayashi-Maskawa (CKM) matrix, but also because of mixing in the squark sector. The latter effect originates from renormalization group scaling with nondegenerate Yukawa couplings. These additional contributions are, however, small enough not to contradict current experimental bounds in a wide range of parameters. The same mechanism may help to solve the problem of CP violation in the system of neutral kaons, but generally results in too large electric dipole moments of leptons and quarks. In particular, for the electric dipole moment of neutron in theories with universal squark masses \tilde{m} one has [26]

$$d_N \simeq 2 \left(\frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \sin \phi \cdot 10^{-23} e \cdot \text{cm}, \quad (8)$$

while the experimental limit is $d_N < 1.1 \cdot 10^{-25} e \cdot \text{cm}$ [27]. Here we denote $\phi = \arg(A_0^* M_{gluino}) - \arg(B_\mu^* \mu M_{gluino})$. So, some additional mechanism is required to suppress the CP violating parameter, $\sin \phi \ll 1$ unless squark masses (in the case of universality, this is true for slepton masses too) are large enough, $\tilde{m} \gtrsim 1 \text{ TeV}$. This is the so-called supersymmetric CP-problem (see Ref. [28] for details).

With the universality assumption, MSSM has only five additional parameters as compared to the Standard Model: besides m_0 , $m_{1/2}$ and A_0 , one has supersymmetric

μ and soft B_μ masses of Higgs fields. The number of parameters may be increased still satisfying the experimental constraints on flavour physics if one replaces the universality condition by the so-called horizontal universality, when different mass parameters m_0^i for different generations are introduced [25].

Another approach exploits the so-called low energy dynamical supersymmetry breaking mediated by gauge interactions. Though its basics have been formulated some time ago [29, 30], these theories underwent intense development in the last few years [31, 32, 33]. The main reason for this fact is the considerable progress in understanding nonperturbative effects which break supersymmetry (see section 5.1 of this review). Mediation itself occurs by means of usual gauge interactions of the visible sector and provides effective superpartner mass scale of order the electroweak scale.

The key advantage of this class of models is a possibility to *calculate* (by usual tools of quantum field theory) supersymmetry breaking parameters of the MSSM. Resulting values of these parameters are such that automatic suppression of flavour violating processes occurs and a small value of electric dipole moment of neutron appears without fine tuning. Yet the theory has very few free parameters, thus it is highly predictive. Electroweak symmetry breaking occurs due to radiative corrections, and to construct unified theories with supersymmetry breaking one has not to invoke nonrenormalizable theories (gravity).

One of shortcomings of models with gauge mediation is their rather complicated structure. Among other problems one might mention the following: 1) in the specific realizations of the mechanism, parameters of the Higgs sector often remain free, and sometimes require fine tuning; 2) in a wide class of models, supersymmetry breaking vacuum is metastable, though its lifetime is much larger than the age of the Universe for a natural choice of parameters; 3) in models with unbroken R-parity, light gravitino is stable leading to cosmological problems; 4) though flavour violation due to squarks and sleptons is naturally suppressed in minimal models, the most general superpotential which respects all MSSM symmetries may contain terms leading to tree level violation of corresponding quantum numbers. We will discuss these questions, among others, in the following sections.

3 Phenomenology of the Minimal gauge mediation model

3.1 Mass spectrum and soft terms

Let us begin with the most general, model-independent features of gauge mediation of supersymmetry breaking. The key ingredient of the mediation mechanism is a set of new fields, messengers, absent in the MSSM but charged under its gauge group. Interaction with a secluded sector results in their supersymmetry breaking spectrum,

namely, in mass splittings in the messenger supermultiplets. Consequently, soft terms are generated in the low energy Lagrangian by radiative corrections due to the MSSM gauge interactions.

Messengers are massive charged fields in vector-like representations of $SU(3) \times SU(2) \times U(1)$. As a rule, they are taken to compose complete multiplets of a unified gauge group, $SU(5)$ for example, in order not to spoil the gauge coupling unification in the MSSM.

Let the messengers Q and \bar{Q} transform as fundamental and antifundamental representations of $SU(5)$, respectively. The simplest way to obtain mass splitting in messenger supermultiplet is to introduce the following Yukawa interaction with a chiral superfield S in the superpotential,

$$W_{ms} = \lambda S Q \bar{Q}. \quad (9)$$

S is MSSM singlet which gets, due to some (not specified yet) dynamics in the secluded sector, nonzero vacuum expectation values $\langle s \rangle$ and $\langle F_s \rangle$ of its scalar and auxiliary components,

$$\langle S \rangle = \langle s \rangle + \langle F_s \rangle \theta^2.$$

Consequently, scalar messengers get tree-level masses,

$$M_{\pm}^2 = \frac{\Lambda^2}{x^2} (1 \pm x), \quad (10)$$

where

$$\Lambda = \frac{\langle F_s \rangle}{\langle s \rangle} \quad \text{and} \quad x = \frac{\lambda \langle F_s \rangle}{\lambda^2 \langle s \rangle^2}. \quad (11)$$

In the physically interesting case³ $0 < x < 1$. Masses of all fermionic components of the messengers are equal to

$$M = \frac{\Lambda}{x}.$$

Interaction with messenger fields results in the soft terms in the MSSM sector. Gauginos get masses at the messenger scale⁴ M through interaction with messengers at one-loop level [29], see Fig. 1,

$$M_{\lambda_i} = c_i \frac{\alpha_i}{4\pi} \Lambda f_1(x). \quad (12)$$

Here $\alpha_1 = \alpha / \cos^2 \theta_W$, α_2, α_3 are coupling constants of weak and strong interactions; c_i is the sum of Dynkin indices of messenger fields running in the loop, $c_1 = 5/3$,

³Otherwise some messenger would have negative mass squared, see Eq. (10), which would result in breaking of electromagnetic and colour gauge groups.

⁴To obtain the physical mass spectrum from Eqs. (12), (13), one has to take into account renormalization group corrections.

$c_2 = c_3 = 1$. Masses depend weakly on x : $f_1(x)$ is a non-decreasing function whose values are close to 1 in the most part of the domain of definition [34]:

$$f_1(x) = \frac{1}{x^2} [(1+x) \log(1+x) + (1-x) \log(1-x)].$$

Two-loop diagrams like one presented in Fig. 1 contribute to masses squared of scalar

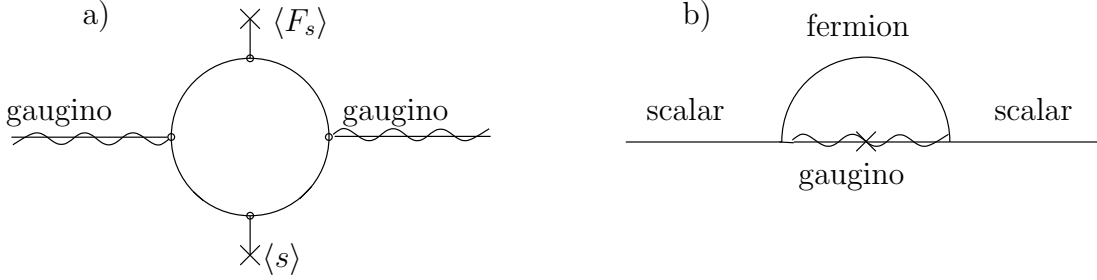


Figure 1: a) Gauginos get masses via diagrams with loops of messenger fields. b) A typical diagram contributing to soft masses of MSSM scalars; a cross denotes the insertion of the loop presented in the figure a).

fields — Higgs bosons, squarks and sleptons [29]:

$$\tilde{m}^2 = 2\Lambda^2 \left[C_3 \left(\frac{\alpha_3}{4\pi} \right)^2 + C_2 \left(\frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} \left(\frac{Y}{2} \right)^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right] f_2(x) = 2\Lambda^2 \sum_{i=1}^3 C_i c_i \left(\frac{\alpha_i}{4\pi} \right)^2 f_2(x). \quad (13)$$

In Eq. (13), C_i are eigenvalues of the quadratic Casimir operator for squark, slepton, or Higgs representation, $C_3 = 4/3$ for colour triplets (zero for singlets), $C_2 = 3/4$ for weak doublets (zero for singlets), $C_1 = \left(\frac{Y}{2} \right)^2$, where Y is the weak hypercharge. Again, $f_2(x)$ is a smooth function close to 1 at the most part of its domain of definition [35]:

$$f_2(x) = \frac{1+x}{x^2} \left[\log(1+x) - 2\text{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\text{Li}_2\left(\frac{2x}{1+x}\right) \right] + (x \rightarrow -x).$$

Let us note that to obtain a phenomenologically acceptable spectrum, one needs a mechanism which generates two nonzero vacuum expectation values, $\langle F_s \rangle$ and $\langle s \rangle$, simultaneously. It is $\langle F_s \rangle$ which results in supersymmetry breaking in the visible sector. The nonzero $\langle s \rangle$ is required to provide (at $x < 1$) positive signs of masses squared of the scalar messengers and to induce chirality breaking insertions in diagrams Fig. 1, thus resulting in nonsupersymmetric contributions to the superpartner masses.

Fermions of the MSSM chiral superfields do not get masses from interaction with messengers because of nonrenormalization theorems [36] (masses of matter fermions,

not being soft terms [9], are not generated when supersymmetry is broken spontaneously). Trilinear coupling, A_0 , appears only at two loop level (see, for instance, Refs. [37, 38]), and its effects are negligible compared to mass terms since it is suppressed by an additional power of $\alpha_i/4\pi$,

$$A_0 \sim \frac{\alpha_i}{4\pi} M_{\lambda_i} \log \frac{M_{\pm}}{M_{\lambda_i}}.$$

In the scheme presented above, which we hereafter refer to as the Minimal Model (MM), superparticle spectrum is given by Eqs. (12), (13). Since $f_1(x)$, $f_2(x)$ are almost constant, all soft masses are determined by a single parameter additional to the Standard Model, that is, supersymmetry breaking scale Λ . One requires $\Lambda \gtrsim 72$ TeV in the Minimal Model [39] in order to satisfy the limits on slepton masses from LEP data.

Since soft masses of particles are proportional to their coupling constants, the heaviest superparticles are squarks ($m_{sq} \sim 1$ TeV at $\Lambda \sim 100$ TeV). The lightest superparticle (LSP) is gravitino, the least interacting superpartner. This kind of LSP is a characteristic feature of models with gauge mediation. Below, we will discuss in detail possible values of gravitino mass and supersymmetry breaking scale which are closely related. The next-to-lightest superpartner (NLSP) in the Minimal model is a combination of $\tilde{\tau}_R$ and $\tilde{\tau}_L$ [39], superpartners of the right- and left-handed τ -leptons. This fact is due to mass splitting caused by a mixing term, $\mu m_\tau \tan \beta \tilde{\tau}_R \tilde{\tau}_L$, which appears in the MSSM Lagrangian after electroweak symmetry is broken. We use the standard notation $\tan \beta = \frac{\langle h_U \rangle}{\langle h_D \rangle}$ for the ratio of the vacuum expectation values of Higgs particles; m_τ is τ -lepton mass. In some extensions of the Minimal model, NLSP may turn out to be neutralino.

Let us emphasize that scalar and gaugino masses are of the same order, both numerically and parametrically, in corresponding couplings α_i .

3.2 Radiative electroweak symmetry breaking in the Minimal model

Consider some characteristics of the electroweak symmetry breaking in the MM. Supersymmetry preserving masses of Higgs particles are given by the μ parameter which was not important in the above consideration. While the soft mass of h_D scalar is given by the general formula, Eq. (13), and is equal to

$$m_{h_D}^2 = \frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 \Lambda^2,$$

mass squared of another Higgs boson, h_U , gets a significant negative contribution due to interaction with \tilde{t} squark, resulting in the leading order in

$$m_{h_U}^2 = m_{h_D}^2 \left(1 - \frac{4Y_t^2}{3\pi^2} \left(\frac{\alpha_3}{\alpha_2} \right)^2 \log \frac{M}{m_{\tilde{t}}} \right), \quad (14)$$

where Y_t is t quark Yukawa constant. In this expression, three-loop contribution is proportional to α_3^2 and is not suppressed compared to two-loop one. It is the three-loop contribution which gives rise to negative $m_{h_U}^2$ resulting in the electroweak symmetry breaking.

To minimize Higgs potential of the MSSM, one should solve the following two equations,

$$\begin{aligned}\mu^2 &= \frac{m_{h_D}^2 - m_{h_U}^2 \tan^2 \beta}{\tan^2 \beta} - \frac{1}{2}m_Z^2, \\ \sin 2\beta &= -\frac{2B_\mu}{m_{h_D}^2 + m_{h_U}^2 + 2\mu^2},\end{aligned}\tag{15}$$

where B_μ is the soft mixing term in the mass matrix of Higgs bosons, Eq. (5). In the MM, this term appears only in the third order of perturbation theory and is suppressed even further due to accidental cancellations. Small mixing in the Higgs sector results in large values of $\tan \beta$. Indeed, at zero mixing, $\tan \beta = \frac{\langle h_u \rangle}{\langle h_d \rangle}$ would be infinitely large since only h_u gets the vacuum expectation value. Note that at large $\tan \beta$, other corrections to Higgs masses become important (for instance, contribution of b quark to h_D mass analogous to Eq. (14) should be taken into account). Accurate calculations [38] give $\tan \beta \sim 50$ in the Minimal model at $\Lambda \sim 100$ TeV.

The value of μ parameter as a function of supersymmetry breaking scale Λ may be extracted from Eqs. (15), with given mass of Z boson. In the MM at $\Lambda \sim 100$ TeV, the value of μ is about 500 GeV. However, the μ parameter is an independent parameter of MSSM Lagrangian which is not a priori related to parameters of the secluded sector. It means that though one succeeds in generating the small scale $\Lambda \ll M_{Pl}$ dynamically, one more free parameter determining the electroweak scale is still present. This fact is the essence of the so-called μ -problem. A few approaches to solve this problem are discussed in section 4.2. From the point of view of phenomenology, their net effect is that B_μ is one more free parameter of the low-energy theory.

3.3 Constraints on the parameters of a secluded sector

Let us consider in more detail parameters of the theory which are related to the secluded sector. The main of them is the supersymmetry breaking scale Λ . As it has been already pointed out, Λ is experimentally bounded from below since it is related to the superparticle masses. The fact that sleptons have not been observed in collider experiments (conservative bound from Z boson width, $\tilde{m}_{sl} > 45$ GeV) together with Eq. (13) mean that $\Lambda\sqrt{n} > 30$ TeV (we take into account the possibility to have n generations of messengers instead of one). To get the correct value of Z boson mass from Eq. (15) without fine tuning of the μ parameter, soft masses of Higgs bosons should not exceed significantly the electroweak scale. The corresponding ‘‘aesthetical’’ upper bound depends on the allowed amount of fine tuning. In any case, one expects that Λ does not exceed a few hundred TeV.

Most upper bounds on the supersymmetry breaking scale deal with the largest F term in the secluded sector, F_{DSB} (as a rule⁵, $F_{DSB} > \langle F_S \rangle$), and with messenger masses $M \sim \langle s \rangle$. These estimates result in conservative upper bounds on Λ since $\Lambda < \langle s \rangle$, $\sqrt{\langle F_S \rangle}$ (see Eq. (11) at $x < 1$).

Bounds on F_{DSB} are mainly related to the gravitino mass $m_{3/2}$ which is non-zero if supersymmetry is broken and which is proportional to $\frac{F_{DSB}}{M_{Pl}^*}$ (hereafter $M_{Pl}^* = M_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass). Namely, once supersymmetry is broken spontaneously, a massless fermion, goldstino, emerges in the spectrum (like massless Nambu–Goldstone bosons emerge due to spontaneous breaking of a Lie group of global symmetry). With gravitational effects taken into account, goldstino is eaten, by means of super-Higgs mechanism, by a longitudinal component of a massless spin 3/2 particle, gravitino \tilde{G} (superpartner of graviton). Longitudinal component of gravitino, \tilde{G}_μ , is expressed in terms of goldstino ψ as $\tilde{G}_\mu = \sqrt{\frac{2}{3}} \frac{1}{m_{3/2}} \partial^\mu \psi$. Resulting gravitino mass is $m_{3/2} = \frac{F_{DSB}}{\sqrt{3}M_{Pl}^*}$.

One of the limits on the scale at which supersymmetry breaking is transferred to the Standard Model follows from requirement of suppression of flavour violating processes. In models of supergravity mediation, such a suppression requires additional assumptions about parameters of the model, as it has been discussed in section 2. Effects of gauge mediation should be much stronger than gravitational ones, so characteristic messenger scale should be considerably lower than Planck scale. To be specific, let soft superparticle mass originated from interaction with messengers be approximately

$$m_{soft} \sim \sqrt{n} \frac{\alpha}{4\pi} \frac{\langle F_S \rangle}{\langle s \rangle} \quad (16)$$

Supergravity contribution to the soft masses is of order gravitino mass, $m_{3/2} \sim F_{DSB}/M_{Pl}^*$. If supergravity contribution is non-universal, then Eq. (16) leads to a typical bound from conditions like Eq. (7),

$$\langle s \rangle \lesssim \sqrt{n} \times 10^{15} \text{ GeV}. \quad (17)$$

This way of reasoning provides another ground for the fact that in models with gauge mediation, gravitino is the LSP.

One more upper bound, the most restrictive up to now, comes from cosmology. It is based on the estimates of influence of NLSP decays on nucleosynthesis [40, 41]. Recall that in gauge mediation, NLSP is either neutralino or τ slepton. Its decays during nucleosynthesis may result in too large amount of light nuclei of deuterium, helium-3, or lithium in the Universe. If NLSP is τ slepton, nuclei are produced due to hadronic decays of τ lepton in the intermediate state. If NLSP is neutralino,

⁵Exact relation between F_S and F_{DSB} depends on the way the secluded sector is incorporated, see section 5.

photons from their decay produce quark-antiquark pairs. To avoid these effects, NLSP should decay before nucleosynthesis begins. One of decay products is goldstino, which interacts with matter via supersymmetric current. The interaction is given [42] by the generalized Goldberger–Treiman formula of the current algebra,

$$\mathcal{L} = -\frac{1}{F_{DSB}} \partial_\mu \psi^\alpha j_\alpha^\mu + h.c. \quad (18)$$

With increasing of F_{DSB} , the interaction becomes less intense, and NLSP lifetime increases. Requirement of NLSP decay before nucleosynthesis thus set an upper bound on F_{DSB} [40],

$$\langle F_{DSB} \rangle \lesssim 10^8 \text{ GeV}. \quad (19)$$

Note that at a special choice of parameters the restriction is not so severe [41]. An assumption of small R parity breaking may relax this bound significantly. Some other cosmological bounds on the supersymmetry breaking scale related to gravitino mass will be discussed in section 6.

3.4 Prospects of the Grand Unification

The possibility to construct realistic supersymmetric Grand Unified Theory (GUT) is one of the most significant arguments in favour of supersymmetric extensions of the Standard Model. Indeed, two principal objections against the minimal non-supersymmetric $SU(5)$ model are difficulties with precise unification of coupling constants and unacceptably fast proton decay. On the other hand, in supersymmetric $SU(5)$ model coupling constants unify with much larger accuracy and proton lifetime is longer due to increase of the unification scale M_{GUT} . Nevertheless, recent high precision measurements of α_s indicate that the same problems are present at the two-loop level in the simplest GUTs based on MSSM (see, e.g., Ref. [43]). More accurate account of the threshold splitting effects at the GUT scale may be one of the possible ways out of these problems.

The situation in gauge mediation models is somewhat worse because of new messenger thresholds. For example, the limits from proton decay do not allow one to obtain exact unification of coupling constants by fine tuning of the triplet Higgs mass in GUTs based on MM as well as its extensions with arbitrary value of $\tan \beta$ or several messenger generations. Another problem is related to the unification of Yukawa couplings at M_{GUT} [44]. This unification is possible only provided $\tan \beta$ is in the narrow ranges near $\tan \beta \simeq 2$ or $\tan \beta \simeq 50 \div 60$.

To invoke messengers which do not compose complete GUT multiplets at low energies is one of the possible ways to solve these problems [45]. A mechanism similar to one which is responsible for doublet-triplet splitting in the Higgs sector (e.g., fine tuning of parameters in $SU(5)$ model) can result in this kind of splitting. Moreover, incomplete messenger multiplets may appear in the framework of more complicated

GUTs. For instance, messengers may carry only down-quark quantum numbers [46]. This kind of spectrum may appear due to the dynamics of the underlying string theory where non-perturbative effects lead to unusually low scale of supersymmetry breaking⁶ $\Lambda \sim 100$ TeV. The resulting masses of weak gauginos are small but experimentally acceptable [48], so this model can be considered as realistic.

The limit from the nucleon decay takes the form $\tan \beta < 10$ at $\Lambda \sim 100$ TeV in the case of incomplete messenger multiplets⁷. Only for three specific sets of messengers $\tan \beta < 17$ [45]. In order to satisfy this limit one should consider one of the extensions of the MM where $\tan \beta$ is an additional free parameter.

To extend the model by introducing new heavy multiplets is another possible solution of the unification problem. For instance, it was demonstrated in Ref. [49] that one can make the limits from $b - \tau$ unification in $SU(5)$ GUT and from $b - t$ unification in $SO(10)$ GUT weaker provided new fields are introduced. This happens due to the breaking of the corresponding symmetry between Yukawa couplings at the unification scale caused by the tree-level mixing of heavy fields with MSSM fields.

4 Extensions of the Minimal model

4.1 The μ problem

As we have already pointed out, some extensions of the Minimal model aim to solve the μ problem and their net effect is that B_μ (or, in another formulation, $\tan \beta$) is a free parameter. In this section, we consider a few approaches to solve the μ problem. Phenomenology of gauge mediation at different values of $\tan \beta$ will be discussed in the next subsection.

Taking into account the relations Eq. (15), one can identify conditions on the theory which solves the μ problem in a natural way [24]:

- One and the same mechanism yields generation of various parameters in the Higgs sector.
- μ term arises in the first order of perturbation theory while B_μ arises only in the second order [50], so μ^2 and B_μ are of equal order, like gaugino and scalar masses.
- No new particles appear at the electroweak scale.
- All new coupling constants are of order one.

In supergravity models, one of the most attractive solutions to the μ problem has been suggested in Ref. [51]. There, a nonrenormalizable mixing $\frac{1}{M_{Pl}} \Phi^+ H_U H_D$ in

⁶Typically, supersymmetry breaking scale in the string inspired models is much higher [47].

⁷ $\tan \beta$ is a relevant parameter here because the rate of $n \rightarrow K^0 \bar{\nu}_\mu$ decay is proportional to $\sin 2\beta$.

the Kahler potential was introduced, where Φ^+ is a hidden sector field with nonzero auxiliary component, $F_\phi \sim m_Z M_{Pl}$. This approach fails in gauge mediation models since it is required there that $F_\phi \ll m_Z M_{Pl}$ so that gauge contribution dominates over gravitational one, see section 3.3.

One may try to obtain a similar solution to the Higgs sector problems in models with gauge mediation. In this case, superpotential

$$W = aZH_U H_D, \quad (20)$$

is introduced, where the vacuum expectation value of the scalar component of Z superfield results in appearance of the μ term. Since one singlet field, S , has been already involved in the model (see Eq. (9)), it seems natural to use it as the field Z [32]. However, in this case $a\langle s \rangle = \mu$ and $a\langle F_s \rangle = B_\mu$, so to satisfy requirements from LEP experiments one should have $\frac{B_\mu}{\mu} = \Lambda \gg m_Z$. This excludes the possibility that $\mu \sim m_Z$ and $B_\mu \sim m_Z^2$ simultaneously. Moreover, from the expressions for messenger masses one gets $(\lambda\langle s \rangle)^2 \gtrsim \lambda\langle F_s \rangle \gg m_Z^2$, so satisfying even a single requirement (either $\mu \sim m_Z$ or $B_\mu \sim m_Z^2$) results in the unnatural condition $\frac{a}{\lambda} \ll 1$. It is possible, of course, to try to explain the smallness of $\frac{a}{\lambda}$ as a consequence of some additional symmetry. Attempts to exploit this approach are presented in Ref. [52] where anomalous global abelian symmetries were discussed which proved useful for explaining the fermion mass hierarchy. In any case, the requirement $B_\mu \sim m_Z^2$ leads inevitably to $\mu \ll m_Z$, so higgsino mass is smaller than the value allowed by experiments. The requirement $\mu \sim m_Z$ contradicts the second equation in (15) because of too large values of B_μ .

It is possible to consider the case when Z and S are different fields, and somewhat modify the interaction, Eq. (20), so that Z components would have appropriate vacuum expectation values. This may be achieved, for instance, by adding a cubic term in Eq. (20) [32, 33] or by introducing an additional interaction with secluded sector fields “symmetric” to the visible sector messengers [24].

All these approaches face several difficulties. A general problem arises, for instance, when global singlet fields are involved. This problem is caused by quadratically divergent tadpoles which appear in two-loop order (even in one loop if the Kahler potential is nontrivial). These terms result in too large vacuum expectation values of the singlets and too large masses of the fields interacting with them.

Four conditions of satisfactory solution to the μ problem mentioned above are fulfilled in the approach with so-called dynamical relaxation mechanism [24, 40] but specific models require many additional fields and interactions to be introduced.

Other ways to solve the μ problem were suggested in several papers, but most of them lead to new serious difficulties. In this sense, elegant or minimal solution is lacking by now.

4.2 Phenomenology at arbitrary $\tan\beta$

At different values of $\tan\beta$, different spectra of superpartners may emerge. Electroweak symmetry breaking results in the generation of D-terms whose contribution to the scalar masses increases with $\tan\beta$. However, Yukawa mixing of left- and right-handed sleptons is proportional to $\tan\beta$. As a result, $\tilde{\tau}_R$ may be lighter than bino. It is $\tilde{\tau}_R$ which is the NLSP at large values of $\tan\beta$ (in the Minimal model with additional free parameter B_μ , for instance, this is the case at $\tan\beta > 25$ [38]).

Right handed τ slepton becomes massless if $\tan\beta$ significantly exceeds 50 [38] which leads to the upper bound on the value of $\tan\beta$. At so large $\tan\beta$, mass squared of the axial Higgs boson may become negative too [53]. Bounds on $\tan\beta$ were discussed in more detail in Ref. [54]. Say, at $x = 0.1$, the absence of light sleptons contributing to Z boson width implies that $\tan\beta < 55$; suppression of tunnelling rate to a vacuum where $U(1)_{\text{EM}}$ is broken results in $\tan\beta < 50$ while absence of such a vacuum gives $\tan\beta < 47$. Lower bounds, $\tan\beta > 1.0 \div 1.5$, follow from the smallness of t quark Yukawa coupling at M_{GUT} , $Y_t(M_{\text{GUT}}) < 3.5$ [53] (this requirement is not necessary, of course). The similar requirement for b quark Yukawa coupling leads to upper bounds on $\tan\beta$ [55]. In unified model with proton decay mediated by triplet Higgses, one has an additional lower bound ($\tan\beta \gtrsim 0.85$ [56]) because proton lifetime $\tau_{\text{proton}} \sim \sin^2 2\beta$.

Let us consider phenomenology of models with gauge mediated supersymmetry breaking at different values of $\tan\beta$ in more detail. As it has already been pointed out, the key difference from other supersymmetric models is that the lightest superparticle is gravitino. The latter interacts with matter not only gravitationally, but, once supersymmetry is broken spontaneously, by interaction of its longitudinal component, goldstino, with supercurrent. The form of the latter interaction can be obtained from Eq. (18) by substituting an explicit expression for supersymmetric current j_α^μ . Integrating by parts, one finally gets the following interaction of goldstino ψ with a chiral multiplet whose scalar and fermionic components have masses m_ϕ and m_χ , and with a vector multiplet (gaugino mass is M_λ),

$$\mathcal{L} = \frac{M_\lambda}{4\sqrt{2}F_{\text{DSB}}} \bar{\psi} \sigma_{\mu\nu} \lambda F_{\mu\nu} + \frac{m_\chi^2 - m_\phi^2}{F_{\text{DSB}}} \bar{\psi} \chi_L \phi^* + h.c. \quad (21)$$

In models of low energy supersymmetry breaking, the inequality $F_{\text{DSB}} \ll M_{\text{Pl}}^2$ is always satisfied, so the interaction Eq. (21) dominates over the pure gravitational one.

The lightest superpartner of the Standard Model particles, that is NLSP, may be either neutralino or slepton (see discussion in section 2, Eqs. (12) and (13)). Since gravitational effects are negligible, the width of NLSP decay to gravitino is described by the equations which may be obtained from the Lagrangian Eq. (21), [57, 58, 55],

$$\Gamma(\tilde{B} \rightarrow \gamma \tilde{G}) = \frac{\cos^2 \theta_w}{16\pi} \frac{m_{\tilde{B}}^5}{F_{\text{DSB}}^2}, \quad \Gamma(\tilde{l} \rightarrow \tilde{G} l) = \frac{1}{16\pi} \frac{m_{\tilde{l}}^5}{F_{\text{DSB}}^2}. \quad (22)$$

In models under consideration, gravitino mass is usually between 1 eV and 1 GeV. Since the interaction Eq. (18) is highly suppressed by F_{DSB} , the most probable process leading to gravitino production is the NLSP decay. Note that $\Gamma(\text{NLSP} \rightarrow \tilde{G} + \text{SM particle})$ is determined by F_{DSB} , see Eq. (22), so by measuring this width one gains information about supersymmetry breaking scale in the secluded sector. Other superparticles decay to Standard Model particles and NLSP via electroweak and strong interactions.

Accelerator phenomenology of models with gauge mediation has been discussed in detail in Ref. [59]. Depending on the parameters of the model, the decay $\text{NLSP} \rightarrow \tilde{G} + \text{SM particle}$ may show up in experiment in different ways [60]. One has three possibilities, the choice being determined by the NLSP mean path,

$$l \approx 5 \left(\frac{100 \text{ TeV}}{\Lambda} \right)^5 \left(\frac{m_{3/2}}{1 \text{ keV}} \right)^2 \text{ m} \quad (\text{NLSP is photino})$$

1. The width is so small that NLSP decays outside the detector, so it is “experimentally stable”. The gravitino mass does not exceed 1 keV in this case [59, 61], so that $\sqrt{F_{DSB}} > 2 \times 10^3 \text{ TeV}$.
2. The width is so large that the corresponding vertex is hard to distinguish from the NLSP production vertex.
3. It is possible to reconstruct the NLSP trajectory from its production to its decay.

Different signatures will correspond to these possibilities, depending on the nature of NLSP. Namely, if NLSP is neutralino, at e^+e^- colliders cases 2) and 3) result in the signature $\gamma\gamma + \cancel{E}_T$, while the first possibility — in missing transverse momentum, \cancel{E}_T . If NLSP is slepton, the case 2) corresponds to the signature $ll + \cancel{E}_T$, and cases 1) and 3) show up as a track of a charged particle much heavier than a lepton. In the latter case, curvature of the track will change abruptly to leptonic one. Since the initial beams leading to NLSP production consist of the usual particles (protons or electrons), the R parity conservation requires even number of superparticles in the final state.

If NLSP is right-handed slepton, a characteristic process in electron–positron collisions is $e^+e^- \rightarrow \tilde{l}_R^+ \tilde{l}_R^- \rightarrow l_R^+ l_R^- + \cancel{E}_T$. Note that four jets production (from hadronic decays of four tau leptons) will be competitive in this case with $e^+e^- \rightarrow \chi_0 \chi_0 \rightarrow 4b + \cancel{E}_T$. The latter process is characteristic to models where NLSP is higgsino [61, 59].

Phenomenology of neutralino (bino or higgsino) decays $\tilde{N} \rightarrow \gamma \tilde{G}$ has been discussed in connection with different experiments: future linear colliders [62], Tevatron [37, 60], electron-photon accelerators [63].

If NLSP is bino, mass and hypercharge difference between right- and left-handed sleptons results in a strong dependence of the cross section from the polarization of

electrons [59],

$$\sigma(e^+e^- \rightarrow \tilde{B}\tilde{B})/\sigma(e^+e^- \rightarrow \tilde{B}\tilde{B}) \simeq 0.01,$$

Besides, cascade processes $e^+e^- \rightarrow \tilde{l}_R^+\tilde{l}_R^- \rightarrow l^+l^-\gamma\gamma + \cancel{E}_T$ would be seen if selectron masses are close to mass of bino.

Proton–antiproton colliders demonstrate another kind of phenomenology. If sleptons are much heavier than bino, then the following processes are possible:

$$p\bar{p} \rightarrow \tilde{W}^\pm\tilde{W}^0 \rightarrow 4j\gamma\gamma + \cancel{E}_T, jjl\gamma\gamma + \cancel{E}_T, l^+l^-l'\gamma\gamma + \cancel{E}_T$$

Processes $p\bar{p} \rightarrow \tilde{B}\tilde{B} \rightarrow \gamma\gamma + \cancel{E}_T$ will be suppressed by large squark masses, while Drell-Yan process will dominate, $p\bar{p} \rightarrow l^+l^-\gamma\gamma + \cancel{E}_T$. The latter occurs either directly or in cascade through bino, depending on the type of NLSP. One event of this type has been discovered at Tevatron by the CDF collaboration [64] with a decay-like kinematics. The possibility to interpret this event in the framework of models with neutralino as NLSP has been suggested in Refs. [59, 61].

The possibility to interpret the $e^+e^-\gamma\gamma + \cancel{E}_T$ event in gauge mediated supersymmetry breaking models where NLSP is either slepton or neutralino has been discussed in detail in Ref. [61]. To make such an interpretation possible, one should have $\tan\beta \simeq 1 \div 3$ in models with neutralino as NLSP while neutralino and higgsino masses have to be in the region $50 \div 100$ GeV. In Ref. [61], fairly narrow regions of MSSM parameters were found where this kind of interpretation is allowed, and other processes to look for at Tevatron and LEP were considered which might refine these regions. Given this explanation of the CDF event, numerous other processes should be seen. In case the evidence for these processes would not be found in current experimental data, the interpretation of an anomalous event is questionable [37, 61]. Note that for $\tilde{\tau}_R$ NLSP, it is unlikely to explain the CDF event in such a way [38]. The event may be explained in the framework of supergravity approach, but this possibility is not very realistic [61] since it requires rather strong fine tuning [65].

Large values of $\tan\beta > 30$ are characteristic to the Minimal Model. To distinguish the models with $\tan\beta > 30$, one may use the processes $b \rightarrow sl^+l^-$ ($l = e, \mu$) whose probabilities are proportional to $\tan^6\beta$. At large $\tan\beta$, the corresponding contribution is 2 to 3 times larger than in the Standard Model [66], so experimental discovery of these processes would be an indirect argument in favour of validity of the Minimal Model of gauge mediated supersymmetry breaking.

4.3 The most general messenger sector

In the Minimal model considered above, the messengers of supersymmetry breaking are Q and \bar{Q} fields in fundamental and antifundamental multiplets of the simplest Grand Unification group, $SU(5)$. In general case, one can have several sets of messengers in different vector-like representations, for instance, $10 + \bar{10}$, $15 + \bar{15}$, 24 for $SU(5)$, $16 + \bar{16}$ for $SO(10)$ etc. Each specific case has certain distinctive characteristics (e.g., fermionic messengers in the adjoint representation may mix with gauginos).

4.3.1 Perturbative unification

Given the requirement of weakness of gauge interactions at M_{GUT} , one has an upper bound on the number of messengers n_r in representations r which is related to their contribution to the β function coefficients. If $\alpha_{GUT} \lesssim 1$, then the effective number of messenger generations $n \equiv \sum_{(r)} n_r c_r$, where c_r are Dynkin indices of the corresponding representations ($c_r = 1$ for the fundamental and $c_r = 3$ for the antisymmetric tensor representation of $SU(5)$), should not exceed four. This means that one can use either four fundamental generations of messengers or one antisymmetric and one fundamental [53] in the $SU(5)$ case, or not more than one generation of $(16 + \bar{16})$ in $SO(10)$. These bounds may be somewhat relaxed for specific relative values of messenger thresholds.

Obviously, messengers should not necessary belong to complete GUT multiplets. This class of models has been considered in Ref. [34]. In that paper, the most general inequalities are presented which are true for any mass and any representation of messengers. Suppose that the fields completing the messenger multiplet to a GUT multiplet have masses of order the GUT scale. Let n_X be the number of messengers carrying the same gauge quantum numbers as the SM fields ($X = L, D, E, U, Q$). Then bounds from the smallness of the MSSM gauge couplings at M_{GUT} result in the following allowed sets of multiplets [34] for messenger masses $M_{\pm} \lesssim 10^7$,

$$\begin{aligned} (n_L, n_D, n_E, n_U, n_Q) &\leq (1, 2, 2, 0, 1) \\ &\leq (1, 1, 1, 1, 1) \\ &\leq (1, 0, 0, 2, 1) \\ &\leq (4, 4, 0, 0, 0) \end{aligned}$$

Generalization of the expressions for soft terms given in section 3.1 to the case of several messenger generations and, perhaps, several singlets S , is straightforward even when mixing among generations is taken into account. Important for phenomenology is that gaugino masses increase as the effective number of messenger generations, n , while the scalar masses increase as \sqrt{n} . Thus slepton is the most probable candidate NLSP in models with a few generations of messengers. Lower bound on Λ is somewhat relaxed in this class of models.

4.3.2 Strong unification

It is not necessary, however, to require the smallness of couplings up to M_{GUT} . In some cases, especially in direct mediation models (see section 5.3), where n is typically large, one may try to relax the constraints coming from perturbative unification since controllable and phenomenologically acceptable unification in the strongly coupled domain may be considered.

The possibility of gauge coupling unification in the strong coupling regime has been considered in the framework of both the Standard Model and its supersymmetric extensions [67]. Recently, this problem attracted interest again [68] after more

precise measurements of the gauge coupling constants at M_Z have been carried out. The latter results differ from the two-loop unification predictions by more than one standard deviation (see, e.g., Ref. [43]).

Note that running gauge couplings of the MSSM α_1 and α_2 increase with energy, so $SU(2) \times U(1)$ is not asymptotically free. These couplings, however, run relatively slow, so Landau poles of these two groups appear at energies higher than the unification scale. Together with the asymptotic freedom of QCD this means that below M_{GUT} all gauge couplings are small, and perturbative analysis is valid. This picture implies the existence of the “desert”, i.e. absence of particles in huge region of masses between superparticle and unification scales. When new particles, like several multiplets of messengers, are introduced, the first coefficients of the β functions increase, so gauge couplings may become large at the unification scale.

Despite the fact that unification in this case occurs at the strong coupling, it is unexpectedly controllable from the low energy point of view, especially in the supersymmetric case [69]. Let us consider one-loop evolution of the coupling constants in an asymptotically non-free unified theory. If M_G is the unification scale and α_G is the value of the unified gauge coupling at that scale, then the renormalisation group equations

$$\frac{d\alpha_i}{dt} = b'_i \alpha_i^2 \quad (23)$$

have a solution

$$\alpha_i^{-1}(Q) = \alpha_G^{-1} + b'_i t,$$

where $t = \frac{1}{2\pi} \log \frac{Q}{M_G}$ and b'_i are the first coefficients of the beta functions of the gauge couplings in the asymptotically non-free theory ($b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$ are the corresponding coefficients of the MSSM). Consider running of the *ratios* of pairs of the gauge couplings. It follows from Eq. (23) that at one loop these ratios have infrared fixed points,

$$\frac{\alpha_i}{\alpha_j} = \frac{b'_j}{b'_i}.$$

These fixed points are reached at the energies which are model-dependent and are determined by the renormalization group flow of ratios of couplings,

$$\frac{\alpha_i(Q)}{\alpha_j(Q)} = \frac{\alpha_G^{-1} + b'_j t}{\alpha_G^{-1} + b'_i t}.$$

The condition that the fixed point is almost reached is $|t| \gg \alpha_G^{-1}/b'_i$. In the case of MSSM, one has $\alpha_G^{-1} \sim 24$, so that the fixed point of α_2/α_1 occurs at $|t| \gg 24$, i.e., at $Q \ll M_G \cdot \exp(-48\pi) \sim 10^{-66} M_G$ which certainly rules out the possibility of the fixed point analysis. However, with new matter added, the situation changes drastically — b_i increase and α_G^{-1} decreases.

In the case of messengers in the complete vector-like representations of the unified gauge group, it is a single number, n , that determines the contribution of messengers to the β functions of all three couplings,

$$b'_i = b_i + n.$$

For $n \geq 5$, the unification occurs at strong coupling. To estimate the energy scale where the ratios of couplings get close to the fixed point values, let us take $\alpha_G = 1$. Then even at $n = 5$, the ratios are almost constant at $Q < 0.04M_G$.

For given n , the threshold corresponding to messenger mass is uniquely determined. Indeed, the low energy running of MSSM couplings is known, and the couplings *should* have the ratios equal to b'_i/b'_j at the threshold. The energy where ratios of MSSM running couplings, determined experimentally at M_Z , reach their fixed point values corresponds to the messenger threshold. Note that at $n \geq 5$ the corresponding thresholds are deep in the region of attraction of the fixed points. For $n = 5$, for example, the threshold is between 1 and 10 TeV, much lower than $0.04 \cdot M_G$. This means that the fixed-point approach is self-consistent. The values of thresholds can be read out from Ref.[69]; values of $6 \leq n \leq 20$ are consistent with current bounds on the messenger mass [70].

The most interesting feature of this scenario is that the strong unification constrains significantly the parameter space of multi-messenger models of gauge mediation. Namely, the mass scale of the messenger fields – one of the two parameters describing the superpartner masses – is determined from self-consistency condition for a given effective number of messengers n . Decreasing the number of parameters results in stronger phenomenological limits on the secluded sector [71, 70].

From the low energy MSSM point of view we just have new boundary conditions for running of the gauge couplings. Instead of requiring the equality of couplings at M_G (as in the case of perturbative unification), one should fix their ratios at the messenger scale. Details of evolution of the couplings near M_G , where they are large, are unknown; however, they do not affect significantly the low-energy predictions [69, 72].

4.4 Messenger-matter mixing

Another possibility to extend the Minimal Model is to introduce messenger-matter mixing [73]. Detailed analysis of the model with mixing can be found in Refs. [74, 75]. Here we consider only its most significant features.

Let us consider a theory with one messenger generation in fundamental representation. Messenger quantum numbers may coincide with quantum numbers either of quarks and leptons or Higgs, depending on R-parity of messengers. In the latter case, triplet messenger fields might give rise to fast proton decay due to possible mixing with Higgs fields, unless the corresponding Yukawa couplings are smaller than 10^{-21}

at messengers masses about 100 TeV [76]⁸.

Let us consider in more detail the case when mixing of the messengers with matter fields is possible. Then doublet components of the Q multiplets have the same quantum numbers as MSSM left leptons and tree-level messenger-matter mixing is possible [73]. It is convenient to introduce common notations for left leptons and doublet messengers,

$$L_{\tilde{i}} = (\tilde{l}_{\tilde{i}}, l_{\tilde{i}}) = \begin{cases} (\tilde{e}_{L\tilde{i}}, e_{L\tilde{i}}) & , \tilde{i} = 1, \dots, 3 \\ (q, \psi_q) & , \tilde{i} = 4 \end{cases}$$

Then the mixing term in the superpotential has the form

$$\mathcal{W}_{mm} = H_D L_{\tilde{i}} Y_{ij} E_j, \quad (24)$$

where $H_D = (h_D, \chi_D)$ is the down Higgs superfield and $E_j = (\tilde{e}_{Rj}, e_{Rj})$ are singlet lepton superfields. Hereafter $\tilde{i}, \tilde{j} = 1, \dots, 4$ label the three left lepton (and right quark) generations and the messenger field, $i, j = 1, \dots, 3$ correspond to the three leptons (quarks) and $Y_{ij}^{(5)}$ are the 4×3 matrices of Yukawa couplings,

$$Y_{ij} = \begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \\ Y_{41} & Y_{42} & Y_{43} \end{pmatrix}.$$

This mixing is natural in the sense that it does not break any MSSM symmetry.

Messenger masses are not a consequence of the Higgs effect in the observable sector. Consequently, overall mass matrices and matrices of Yukawa couplings are not proportional to each other. It is convenient to integrate out heavy messenger fields in order to analyse low energy processes with lepton flavour violation. One should first check, however, that these processes are suppressed at the tree level in the model with mixing (24).

Let us diagonalize mass matrix of fermion fields (including left and right messenger fermions) in order to explore tree level mixing.

In principle, non-diagonal terms in the corresponding rotation matrices can lead to lepton flavour violation through one loop diagrams involving sleptons and gaugino in analogy to models with gravity mediation [80]. These terms in the model with mixing are highly suppressed by the seesaw mechanism. Even at $Y_i \sim 1$ the corresponding rates of lepton decays are too small to be observable in current experiments.

⁸It is worth noting that a possibility to identify messengers and Higgs fields has been discussed [77, 78]. The main problem of this approach is related to triplet Higgs fields. Namely, if they are light then it is highly non-trivial (but still possible in some cases) to suppress fast proton decay while heavy triplet messengers lead to very light gluinos which is hardly acceptable from phenomenological point of view. Light gluino problem also arises if one identifies messengers with Higgs particles responsible for intermediate symmetry breaking, $SU(2)_L \times SU(2)_R$ to $SU(2)_L \times U(1)_Y$ [79].

Mixing in scalar sector is also negligible at the tree level. After the scalar messengers are integrated out at the tree level, the lepton flavour violating terms in the mass matrix of right sleptons are of order

$$Y_i Y_j (v_D^2 x^2 + \frac{\mu^2 v_U^2 x^2}{\Lambda^2}) \quad (25)$$

for generic values⁹ of x . These terms are smaller than the one loop contributions (see below) at $\Lambda \gtrsim 10$ TeV. Mixing terms in the left slepton sector are even less significant.

Mixing appears in the quadratic as well as in the interaction part of the effective Lagrangian after messengers are integrated out. Let us find the leading term in large parameter Λ . The contribution to the lepton mixing matrix is proportional to the vacuum expectation value of the corresponding Higgs field. Contribution of the order Λ due to the inertactions of SM fermions with messengers is forbidden because of chirality (see section 3.3). So large mixing can appear only in the slepton sector. With superpotential Eq. (24), corresponding terms appear in the mass matrix of right sleptons due to their interactions with the Higgs and messengers,

$$\delta m_{ij}^2 = -\frac{1}{16\pi^2} \frac{\Lambda^2}{x^2} \left\{ -\log(1-x^2) - \frac{x}{2} \log\left(\frac{1+x}{1-x}\right) \right\} Y_{4i}^* Y_{4j}. \quad (26)$$

Let us note that the term (26) is larger than the tree-level mixing of right sleptons. At small x , Eq. (26) has the form,

$$\delta m_{ij}^2 \approx -\frac{x^2}{6} \frac{1}{16\pi^2} \Lambda^2 Y_{4i}^* Y_{4j}.$$

One-loop contribution exceeds the tree-level one provided the following inequality holds (see Eq. (26)),

$$v_D^2 + \frac{\mu^2 v_U^2}{\Lambda^2} < \frac{1}{6} \frac{1}{16\pi^2} \Lambda^2. \quad (27)$$

At $\Lambda > 10$ TeV and typical values of $\mu \approx 500$ GeV, the relation (27) is indeed satisfied. The contribution of Eq. (26) to \tilde{m}_R^2 is negative so that the positivity of slepton mass squared results in theoretical limits on Yukawa couplings Y_{4i} [73].

Let us consider the role of mixing in low-energy lepton physics. It was shown in Ref. [74] that the rates of lepton flavour violating processes are not too large in the natural range of parameters. For instance, the current experimental limit on the branching ratio of $\mu \rightarrow e\gamma$ decay $\text{Br}(\mu \rightarrow e\gamma) < 4.9 \cdot 10^{-11}$ [48] gives for the mixing parameters,

$$\sqrt{Y_{41} Y_{42}} x \lesssim 3.0 \times 10^{-3} \quad (28)$$

⁹Hereafter, by generic values of x we understand x not very close to 1. Only at $x \approx 1$ some numerical coefficients slightly change in the following equations.

It is worth noting that parameters Y and x appear in the combination Yx in this inequality. However, this fact does not imply that Y can be arbitrarily large provided x is small enough. At the values of x of order gauge coupling constants, mixing in gauge vertices due to the loops involving messengers and Higgs particles is significant [75]. This sets a limit on $Y\sqrt{\alpha_i}$.

Similar mechanisms are responsible for flavour violating τ decays, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. Corresponding limits follow from the current bounds on the branching ratios of these decays,

$$\text{Br}(\tau \rightarrow e\gamma) < 2.7 \cdot 10^{-6} \quad \text{and} \quad \text{Br}(\tau \rightarrow \mu\gamma) < 3.0 \cdot 10^{-6}.$$

Messenger-matter mixing is possible in quark sector as well, because some of the messengers have quantum numbers of d -quarks. This mixing leads to additional contributions to FCNC processes such as $K^0 - \bar{K}^0$ mixing, which happen in the Minimal Model due to CKM matrix. Generally speaking, the corresponding Yukawa coupling is not related directly to lepton couplings Y_{ij} from (24). However, the quark-messenger mixing constants also do not exceed 10^{-3} which is compatible with possible unification.

Let us note that messenger-matter mixing leads to the negative contributions of Higgs doublet masses squared [73]. It can significantly alter electroweak breaking in the theory. For instance, in the case of fundamental messenger representations, the limits on the Yukawa couplings Y allow the contribution to $m_{h_D}^2$ to be large enough to make $\tan\beta \sim 1$. At these low values of $\tan\beta$, NLSP is neutralino so that phenomenology can be rather different from the MM case.

Similar mixing is possible for other messenger representations as well. For example, in the case of messengers in antisymmetric representations of $SU(5)$ group, the limits on the mixing parameters coming from flavour physics are somewhat weaker than for fundamental messenger representations at the same value of Λ .

Finally, let us note that messenger-matter mixing leads to the absence of cosmological problems related to stable charged (and coloured) particles (messengers) (see section 6.2). The solution of this problem due to tree level mixing is one of the most natural approaches.

5 Towards incorporation of the secluded sector

We have discussed so far the model-independent aspects of gauge mediated supersymmetry breaking. All significant information about the secluded sector was contained in parameters Λ , x and supersymmetric Higgs mass μ . However, the combined analysis of the effects both in the observable and in the secluded sector is one of the essential advantages of gauge mediation models As compared to gravity mediation scenario. In this section we consider specific models incorporating secluded sector, where dynamical supersymmetry breaking (DSB) takes place.

5.1 Non-perturbative dynamics of supersymmetric gauge theories

The recent revival of interest in construction of realistic models with low energy supersymmetry is in great part due to developments in the study of supersymmetric theories. On the one hand, these methods allow to enlarge significantly the number of theories exhibiting DSB and, on the other hand, they provide powerful tools for the detailed analysis of the consequences of DSB. We concentrate on the recent achievements in the study of non-perturbative effects in supersymmetric theories before the consideration of the specific models. The earlier results are discussed in detail in Refs. [81, 82, 83].

5.1.1 Flat directions: classical theory

The presence of the classical flat directions — submanifolds of the space of fields which consist of, generally speaking, physically different supersymmetric minima of the tree level potential — is the characteristic feature of supersymmetric theories. Eq. (3) implies that the classical vacuum configuration satisfies the following conditions,

$$D^a = 0, \quad F = 0. \quad (29)$$

Usually, in order to solve Eqs. (29), one first finds field configurations with $D^a = 0$, and study whether they satisfy the condition of stationarity of the superpotential. The second stage is purely technical and does not present fundamental difficulties usually. That is why we will first consider the case $W = 0$. One can find in literature the explicit forms of vacuum configuration in a wide number of specific models, but a universal method to obtain the solutions of (29) is missing. As it has been pointed out already in Ref. [81], the dynamics along the flat direction is conveniently described in terms of holomorphic gauge invariants composed of the matter fields. The following useful theorem was formulated and proven in Ref. [84].

Theorem.

The classical space of vacua is isomorphic to the space of the constant field configurations, which cannot be transformed one to another by the action of the complexified gauge group. Holomorphic gauge invariants composed of the matter fields are the natural coordinates on this space.

To prove this statement, gauge fixing conditions with additional (in comparison to Wess-Zumino gauge) symmetry were used in Ref. [84]. The field V has the following form in this gauge,

$$V = C - \theta\sigma^\mu\bar{\theta}v_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D.$$

Then the theory is invariant under the group G_c , which is complexification of the group G . One can show now that for every constant field configuration it is possible

to find the value of the field C in such a way that the vacuum energy is equal to zero. This condition can be written in the following form in this gauge,

$$\frac{\partial}{\partial C} \phi_i^\dagger e^C \phi_i = 0 .$$

Every field configuration which is obtained in this way can be uniquely (up to a usual gauge transformation) transformed to the configuration with $C = 0$ by the action of G_c . This means that flat directions are in fact equivalent to the quotient space of constant fields ϕ with respect to the group G_c . The corresponding manifold is described in terms of the full set of invariants of this action. Holomorphic gauge invariant polynomials form this set.

This theorem does not provide us with the explicit form of the classical flat directions in terms of elementary fields. However, description of the space of vacua through gauge invariants is convenient in the quantum theory, where these invariants become effective low energy degrees of freedom (moduli). Generally speaking, taking into account quantum corrections to the classical flat directions is necessary for the complete analysis of the vacuum structure. Let us consider this point now.

5.1.2 Flat directions: quantum theory

Typically, quantum corrections change the vacuum structure of the theory. Non-renormalization theorems imply that quantum flat directions coincide with the classical ones within the perturbation theory (abelian theory with nonzero sum of charges of matter fields is the only exception). But these theorems tell nothing about non-perturbative corrections which can alter the classical consideration. A classical direction can disappear completely or in part because of the generation of non-perturbative superpotential, can change its geometry, or remain the same but describe the theory which is completely different from the classical one. Finally, even the absence of classical flat directions can be very important in the full quantum theory.

Practically all these possibilities are realized in the simplest supersymmetric gauge theory — Supersymmetric Quantum Chromodynamics (SQCD). It contains N_F flavours of chiral superfields of (anti-)quarks $(\bar{\Phi}_i), \Phi_i$ in the (anti-)fundamental representation of the gauge group¹⁰ $SU(N_C)$. The Lagrangian is given by Eq. (2) with the superpotential

$$W = \sum_1^{N_F} m_i \Phi_i \bar{\Phi}_i . \quad (30)$$

Let us consider the dynamics of this theory at different values of N_F . Supersymmetry implies holomorphy of superpotential as function of chiral superfields and

¹⁰In what follows all formulas will be presented for the case $N_C > 2$. The picture remains qualitatively the same for $N_C = 2$, but concrete formulas can change due to the fact that the fundamental representation of $SU(2)$ is isomorphic to its conjugate.

coupling constants [85, 86]. This requirement combined with the invariance under non-anomalous symmetries is a powerful tool allowing to find out the exact form of the superpotential in many cases. Also, it might occur useful to find out the superpotential in some limiting cases, when the theory gets simplified.

$$\mathbf{N}_F = \mathbf{0}$$

We are dealing with Yang-Mills theory with one adjoint fermion (superpartner of gauge boson — gaugino) in this simplest case. There are no flat directions at the classical level, but non-trivial quantum effects are present. Namely, R -symmetry is broken by the gaugino condensate.¹¹ This condensation leads to the effective superpotential

$$W_{eff} = c\Lambda^3 . \quad (31)$$

As usual, $\Lambda = \mu e^{\frac{2\pi i\tau(\mu)}{3N_C}}$ is the scale where the gauge dynamics becomes non-perturbative and c is some constant. One can obtain the expression (31) from symmetry considerations if τ is treated as an external field [87] transforming as follows,

$$\tau \rightarrow \tau + \frac{N_C}{\pi}\alpha$$

under R -symmetry (which becomes non-anomalous then). The meaning of the superpotential (31) is that the value of gaugino condensate is given by

$$\langle \lambda\lambda \rangle = 16\pi i \frac{\partial}{\partial \tau} W_{eff} = -\frac{32\pi^2}{N_C} c \mu^3 e^{\frac{2\pi i\tau(\mu)}{N_C}} . \quad (32)$$

There are N_C vacua with different phases of the gaugino condensate. A surprising fact is that it is absolutely unclear which diagrams contribute to the condensate (32). Such a Green's function corresponds to the transition with the change of the topological charge at $\frac{1}{N_C}$ and cannot be obtained by means of instanton calculations.

The parameter Λ becomes a function of dynamical fields in more complicated theories and the superpotential (31) generates additional terms in the scalar potential. An interesting application of this mechanism is described below (see section 5.3.1).

$$\mathbf{0} < \mathbf{N}_F < \mathbf{N}_C$$

In this case, at zero quark masses, there are classical flat directions parametrised by the set of “meson” gauge invariants,

$$M_{ij} = \Phi_i \bar{\Phi}_j . \quad (33)$$

The effective superpotential which is invariant under all non-anomalous symmetries of the theory has the following form [81],

$$W_{eff} = c \left(\frac{\Lambda^{3N_C - N_F}}{\det M} \right)^{\frac{1}{N_C - N_F}} . \quad (34)$$

¹¹It is worth mentioning, that in the absence of the matter fields the continuous R -symmetry is broken by anomalies to its discrete subgroup Z_{2N_C} . It is the latter one which gets broken down Z_2 by gaugino condensate.

The superpotential (34) lifts all classical flat directions and there appear “run-away” vacua at the infinite values of squark fields. At non-zero quark masses there are again N_C different vacua at finite values of the matter fields with the gauge group broken to $SU(N_C - N_F)$. The chiral symmetry is broken by the gaugino condensate, which is related to the squark condensate through the Konishi anomaly (see, e. g., Ref. [85]).

The mechanism of generating the superpotential (34) depends on the number of flavours N_F . At $N_F = N_C - 1$ one can explicitly obtain the value of the constant c in Eq. (34) by calculating the one-instanton Green’s function [81, 83, 85], but at smaller N_F the superpotential is generated due to gaugino condensation in the low energy theory. If one knows W_{eff} at some value of N_F then it is straightforward to obtain W_{eff} for $N_F - 1$ quark flavours. One should give the mass m to one of the quarks and take the limit $m \rightarrow \infty$. The value of Λ in the low energy theory can be obtained from matching of the coupling constants at the decoupling scale m . This procedure serves as a non-trivial check of the results.

$$\mathbf{N}_F = \mathbf{N}_C$$

In addition to meson gauge invariants (33), there are baryon B and antibaryon \bar{B} invariants in this case (we present here their general form for arbitrary $N_F \geq N_C$; only one pair of baryon and antibaryon is present at $N_F = N_C$),

$$B^{i_{N_C+1} \dots i_{N_F}} = \epsilon^{i_{N_1} \dots i_{N_F}} \Phi_{i_1} \cdot \dots \cdot \Phi_{i_{N_C}} \quad (35)$$

and analogously for antibaryons. One can check explicitly that at the classical level these invariants satisfy the following constraint

$$B\bar{B} - \det M = 0. \quad (36)$$

The classical space of vacua exhibits singularities. They correspond to the appearance of additional massless degrees of freedom at some points of the moduli space. These are gluons of the unbroken gauge group. At the quantum level, all singularities turn out to be smoothed out and the gauge group is broken in the entire moduli space [88]. The mechanism responsible for this phenomenon is illustrated in Fig. 11. Formally, the classical constraint (36) takes the following form in the quantum theory

$$\det M - B\bar{B} = \Lambda^{2N_C} \quad (37)$$

(vacuum expectation values (vev’s) of the corresponding fields are understood on the left hand side of this equation). To check it one can add the sources, $W_{tree} = \text{Tr} m M + b B + \bar{b} \bar{B}$ in order to find the vev’s of the corresponding operators and take the limit of zero sources. It is possible to obtain all points of the manifold (37) by varying the ways of taking this limit. For instance, the Lagrangian of massive SQCD is obtained if $b = \bar{b} = 0$ from the very beginning. Selection rules and exact instanton calculations give the following vev’s of the gauge invariants (33) and (35) in this theory,

$$M_{ij} = \Lambda^2 (\det m)^{\frac{1}{N_C}} m_{ij}^{-1},$$

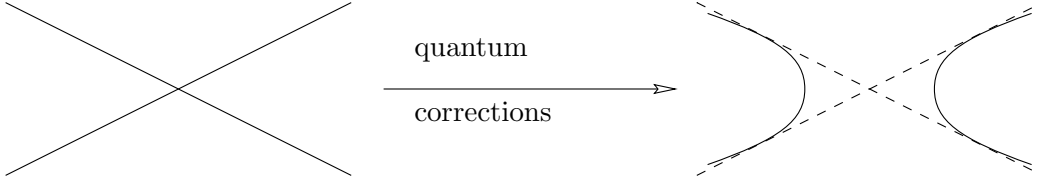


Figure 2: Quantum deformation of the classical moduli space.

$$B = \bar{B} = 0 .$$

These vev's indeed satisfy Eq. (37).

This result can be also related to the description of SQCD at smaller number of flavours. To do this, one should give mass to one of the quark fields and exclude this field by making use of the constraint (37).

The phenomenon just described is called deformation of the moduli space and may lead to supersymmetry breaking.

$$\mathbf{N}_F = \mathbf{N}_C + \mathbf{1}$$

Mesons and baryons (33), (35) describe the classical flat directions in this case as well, but they satisfy three different constraints,

$$B_i \bar{B}_j - \det M (M^{-1})_{ij} = 0 \tag{38}$$

$$M_{ij} B_i = M_{ij} \bar{B}_j = 0 .$$

The main difference with the previous case is that the classical constraints (38) are still valid at the quantum level. One can check this by introducing sources in a similar as in the case $N_F = N_C$. So we are dealing with the quantum moduli space which exactly coincides with the classical one. Classical singularities are not smoothed out here, but they are present at the quantum level as well. However, their physical meaning is different. In both cases their presence is related to the additional massless particles in the spectrum. At the classical level, these particles are gauge bosons of the unbroken group. This cannot be the case in quantum theory, because the dynamics is non-perturbative and colourless degrees of freedom are expected.

Let us consider a singular point $M = B = \bar{B} = 0$ for example. The fact that all global symmetries of the theory are unbroken at this point indicates that all mesons (33) and baryons (35) are massless there. A powerful tool to check the conjectures of this type is

the 't Hooft anomaly matching conditions [89]:

Anomalies calculated with elementary and composite degrees of freedom should coincide for all global symmetries of the theory that are not broken spontaneously.

The proof of this statement deserves to be presented here because of its simplicity and elegance. Let us consider a theory with the gauge group G_C and global symmetry group G_F . Let us extend the gauge group to $G_C \times G_F$, in such a way that dynamics of G_F will not alter the dynamics of G_C . Then, besides additional gauge bosons, one should add massless fermions charged only under G_F in order to obtain non-anomalous theory with weakly coupled group G_F at low energies. In the infrared region the theory is described by the same composite fields as the original theory and by the new fields which are neutral under G_C and, consequently, are not involved in the non-perturbative dynamics of this group. The group G_F is non-anomalous at low energies as well. Then the fact that the additional fields give the same anomalies at high and low energies implies that composite degrees of freedom contribute to the anomalies in the same way as elementary fields of the theory.

The 't Hooft matching conditions are meaningful if the matrix elements of the divergences of the corresponding currents are well defined both at high and low energies. For instance, the abelian axial current in ordinary QCD does not satisfy this criterion, because its divergence is related to the field strength of gluons, which is not well defined in the non-perturbative region. This implies that there is no axial symmetry in the low energy theory and anomaly matching is meaningless.

Certainly, matching of anomalies is not the proof of validity of a given effective description of the theory under consideration, but it serves as a highly non-trivial check¹².

In the case of SQCD with $N_F = N_C + 1$, the non-anomalous group of global symmetries is $SU(N_F)_L \times SU(N_F)_R \times U(1)_B \times U(1)_R$. Quarks have the following quantum numbers with respect to this group,

$$\Phi = (N_F, 1, 1, \frac{1}{N_F}), \quad \bar{\Phi} = (1, \bar{N}_F, -1, \frac{1}{N_F}),$$

and composite degrees of freedom are

$$M = (N_F, \bar{N}_F, 0, \frac{2}{N_F})$$

$$B = (\bar{N}_F, 1, N_F - 1, \frac{N_F - 1}{N_F}), \quad \bar{B} = (1, \bar{N}_F, -N_F + 1, \frac{N_F - 1}{N_F}).$$

It is straightforward to check that contributions to anomalies from fundamental and composite particles are equal. Let us consider, for example, Green's function of $SU(N_F)_L^2 \times U(1)_R$ currents. At high energies the contribution to its anomaly is equal to $N_C d^{(2)}(N_F)$ (we leave only group factor, and skip the factor which is identically the same for all Green's functions), $d^{(2)}(N_F)$ is the Dynkin index of quark representation.

¹²Recently [90] an example of the model has been presented where the 't Hooft conditions are satisfied but the supposed effective theory does not provide a correct description of the low energy dynamics.

In the effective theory, the anomaly is saturated by baryons and it is equal to

$$R(B)d^{(2)}(\bar{N}_F) = (N_F - 1)d^{(2)}(N_F) = N_C d^{(2)}(N_F) .$$

In a similar way, one can check the anomaly matching for other Green's functions.

The correct description of low energy excitations about the vacua (38) is given by the effective superpotential

$$W_{eff} = \frac{1}{\Lambda^{2N_F-3}} (B_i M_{ij} \bar{B}_j - \det M) , \quad (39)$$

which is invariant under all non-anomalous symmetries. The condition of its stationarity implies the constraints (38); the superpotential of Eq. (39) describes massless fields at the origin, while at the large vev's some of the fields become massive, which corresponds to their explicit elimination by making use of constraints (38). Finally, integrating out one of the quarks leads to the correct results at $N_F = N_C$.

$$\mathbf{N}_F > \mathbf{N}_C + \mathbf{1}.$$

The theory becomes more and more weakly coupled in the infrared when the number of flavours increases. At $N_F > 3N_C$ asymptotic freedom disappears and the low energy theory describes non-interacting quarks and gluons. The situation is much more interesting in the intermediate region $N_C + 1 < N_F < 3N_C$. In similarity to the case $N_F = N_C + 1$, classical flat directions coincide with the quantum moduli space and are described by the invariants (33), (35) satisfying the constraints, which are straightforward generalizations of Eq. (38). However, an attempt to describe the behaviour of the theory in analogy to the previous case fails. This follows, for instance, from the failure of anomaly matching. Another manifestation of the same problem is that the superpotential,

$$W_{eff} = \frac{1}{\Lambda^{2N_F-3}} (B_{i_1 \dots i_{N_C-N_F}} M_{i_1 j_1} \dots M_{i_{N_C-N_F} j_{N_C-N_F}} \bar{B}_{j_1 \dots j_{N_C-N_F}} - \det M) , \quad (40)$$

which is a generalization of Eq. (39) and leads to the classical constraints between mesons and baryons as consequences of the equations of motion, does not have R -charge equal to two, as is required by R -symmetry.

On the other hand, it has been argued already in Ref. [91] that in the usual QCD with a certain relation between N_F and N_C , non-trivial infra-red fixed points can be present. β -function in the supersymmetric case has the form, [92]

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi^2} \frac{3N_C - N_F(1 - \gamma(\alpha))}{1 - N_C \frac{\alpha}{2}} ,$$

where

$$\gamma(\alpha) = -\frac{\alpha}{2} \frac{N_C^2 - 1}{N_C} + \mathcal{O}(\alpha^2)$$

is the anomalous dimension of the quark field. It is straightforward to check that in the limit of large N_F and N_C , with $N_C \alpha$ and $3 - \frac{N_F}{N_C} \ll 1$ kept constant, β -function indeed has a non-trivial zero in the perturbative region. Consequently, low

energy dynamics is described by the superconformal theory of interacting quarks and gluons. It was suggested by Seiberg [93] that this is the case in the entire region $3N_C/2 < N_F < 3N_C$ at arbitrary (not necessary large) N_C and N_F .

The dimension of the chiral operator M_{ij} is smaller than one at $N_F < 3N_C/2$, that is M_{ij} cannot correspond to any unitary representation of the superconformal algebra. At the boundary value $N_F = 3N_C/2$ mesons have dimension one and describe free fields. The conjecture [93] is that from this value of N_F down to $N_F = N_C + 2$ all low energy degrees of freedom are free and they are described by another effective gauge theory, called “magnetic” (dual to the original, electric) theory. Baryons of the electric theory have $N_F - N_C$ indices and occur to be composite states of $N_F - N_C$ “magnetic” quarks $\tilde{\Phi}$. The significant difference of this case from all the previous ones is that low energy theory is the gauge theory, but with new gauge group $SU(N_F - N_C)$. Magnetic quarks $\tilde{\Phi}$ and antiquarks $\bar{\tilde{\Phi}}$ are charged under this “magnetic” gauge group and fall in the following representations of the global symmetries group (which should be the same both in the electric and magnetic theories),

$$\tilde{\Phi} = (\overline{N_F}, 1, \frac{N_C}{N_F - N_C}, \frac{N_C}{N_F}), \quad \bar{\tilde{\Phi}} = (1, N_F, -\frac{N_C}{N_F - N_C}, \frac{N_C}{N_F}).$$

Magnetic theory is infrared free at $N_F < 3N_C/2$. At $3N_C/2 < N_F < 3N_C$, magnetic theory (as well as electric one) has non-trivial fixed point; both magnetic and electric theories represent two descriptions of the same superconformal theory in different terms. Dual theory becomes non-perturbative at larger number of flavours where electric theory gives the correct description of the dynamics. Mesons (33) appear in the dual theory as independent neutral fields interacting with the dual quarks through the superpotential

$$W_d = \tilde{\Phi} M \bar{\tilde{\Phi}}. \quad (41)$$

Its role is to break extra $U(1)$ symmetry acting on the meson fields and to ensure the proper correspondence between flat directions in the electric and magnetic theories.

In this description all ’t Hooft anomaly matching conditions are satisfied. Moreover, the conjecture satisfies other tests such as simultaneous deformation of the dual theories and integrating out a flavour. Furthermore one can dualize the theory twice and obtain the original theory, as it is natural to expect (at first sight there appear two additional singlets corresponding to meson fields; but one can integrate out one of them by making use of Eq. (41) and find that the second one is expressed through the quark fields according to Eq. (33)). Finally, this duality can be related to $N = 2$ and string dualities [94]. To conclude this section, let us note that most of the presented results can be generalized to the case of other simple groups [95, 96].

5.2 Models with mediation via singlet field

We discuss specific models incorporating secluded sector in the remainder of this section. We analyze several typical examples illustrating different mechanisms of

generating non-zero $\langle s \rangle$ and $\langle F_s \rangle$ and of dynamical supersymmetry breaking. Let us consider first the theories where Yukawa-type interaction $SQ\bar{Q}$ is the only interaction between messengers and secluded sector. So called models of direct mediation, where messenger fields are charged under gauge group of the secluded sector are discussed in section 5.3.

5.2.1 Models with the additional $U(1)_m$ gauge group

We start with the discussion of the Dine-Nelson models [31, 32, 33]. These are the models, which attracted considerable interest to low energy supersymmetry breaking recently. As a rule, supersymmetry breaks down spontaneously due to non-perturbative dynamics of the gauge group of the secluded sector. Supersymmetry breaking dynamics of the secluded sector and a mechanism that generates non-zero $\langle s \rangle$ and $\langle F_s \rangle \lesssim \langle s \rangle^2$ are two basic components of the full theory. The characteristic feature of the Dine-Nelson models is the presence of a weakly coupled $U(1)_m$ gauge group which does not affect supersymmetry breaking.

New intermediate sector is incorporated in order to generate non-zero $\langle s \rangle$ and $\langle F_s \rangle \lesssim \langle s \rangle^2$. Fields of this sector interact with the singlet and do not affect non-perturbative dynamics.

There is no interaction of these fields with the secluded sector in superpotential. On the other hand, these fields are charged under $U(1)_m$ group and obtain information about supersymmetry breaking through non-zero $U(1)_m$ D -terms. The latter appear because some of the secluded sector fields are also charged under $U(1)_m$. If this group is unbroken in the supersymmetry breaking vacuum, then the interaction of the charged under $U(1)_m$ fields Z_{\pm} with singlet S of the type

$$W = \lambda_1 Z_+ Z_- S + \frac{\lambda_3}{3} S^3$$

leads to the generation of non-zero $\langle s \rangle$ and $\langle F_s \rangle$.

Supersymmetry breaking sector considered in Ref. [31] is rather complicated (due to the so called R-axion problem). We will consider the model of Ref. [32] where the secluded sector is much simpler. It is based on the well-known “3-2” model [82], which we will describe now.

“3-2” model. The gauge group of this model is $SU(3) \times SU(2)$ with the following set of the matter fields (numbers in parentheses denote $SU(3) \times SU(2)$ representation of the corresponding field),

$$\begin{aligned} & \text{one field } Q \text{ } (3, 2), \\ & \text{two fields } \bar{L}_I \text{ } (\bar{3}, 1), \text{ } I = 1, 2, \\ & \text{one field } \bar{R} \text{ } (1, 2) . \end{aligned}$$

The following renormalizable superpotential, compatible with all symmetries of the theory (global R-symmetry and gauge symmetries) can be introduced,

$$W = kQ\bar{L}_1\bar{R} \quad (42)$$

(k is the corresponding Yukawa coupling). Let us discuss dynamical supersymmetry breaking in this model along the line of Ref. [97].

The complete set of holomorphic invariant polynomials in this model is

$$X_I = Q\bar{L}_I\bar{R}, \quad Y = (Q\bar{L}_1)(Q\bar{L}_2).$$

Consequently, classical flat directions form 3-dimensional complex manifold with coordinates X_I , Y in the absence of the superpotential. The situation changes when the superpotential (42) is included. Let us first consider the condition of stationarity of $W(\Phi)$ along the direction \bar{L}_I ,

$$\frac{\partial W}{\partial \bar{L}_1^a} = kQ^a\bar{R} = 0. \quad (43)$$

Upon multiplying this equation by \bar{L}_I^a and contracting the group index a , one obtains that $X_I = 0$. In analogy, stationarity along the direction \bar{R} ,

$$\frac{\partial W}{\partial \bar{R}^i} = kQ^i\bar{L}_1 = 0$$

implies that $Y = 0$. Hence all flat directions are lifted in the presence of superpotential (42), and the only vacuum in “3-2” model is the origin of the field space. This observation indicates that in fact (*i.e.* with quantum non-perturbative corrections taken into account) supersymmetry might be broken. Indeed, the following simple criterion of supersymmetry breaking was suggested in Ref. [82],

Supersymmetry is broken if non-compact flat directions are absent and some global symmetry is spontaneously broken in the theory.

This statement is related to the presence of massless *real* scalar field (Goldstone boson) in the theory with spontaneously broken global symmetry. Let us suppose that supersymmetry is unbroken. Then this field should have one more massless scalar partner in order to form a chiral multiplet which includes *complex* scalar field. While the usual Goldstone boson corresponds to the action of the broken group generator on the vacuum, its partner corresponds to the action of complexified generator (the moduli space in supersymmetric theory is invariant under the complexification of the group of global symmetries due to holomorphy of the superpotential). The orbits of the action of the complex group are not compact, so that this field parametrizes a non-compact flat direction, in contradiction to the assumption of the theorem.

Global R-symmetry is present in “3-2” model. This symmetry is unbroken only if all vev’s of the matter fields are equal to zero. The above criterion implies that in order to check that supersymmetry is broken in this case, it is sufficient to demonstrate that

the origin of the field space cannot be the vacuum of the theory after all quantum corrections are taken into account. The specific mechanism of generating non-zero vev's depends on the ratio of the dynamical scales Λ_2 and Λ_3 of $SU(2)$ and $SU(3)$ groups, respectively.

$\Lambda_2 \ll \Lambda_3$. In this case low energy dynamics is completely determined by $SU(3)$ group. There are two flavours in the fundamental ($SU(2)$ -components of the field Q) and in antifundamental (\bar{L}_I) representations of this group. So, we are dealing with the specific case of SQCD with $N_F = N_C - 1$. As discussed above, one-instanton superpotential

$$W_{ins} = \frac{\Lambda_3^7}{Y}. \quad (44)$$

is generated in this case. The superpotential (44) is singular in the origin, so this point cannot represent the vacuum of the theory and supersymmetry is indeed broken. So, the “3-2” model in this limit is probably the simplest example of supersymmetry breaking by means of the generation of non-perturbative superpotential.

$\Lambda_2 \gg \Lambda_3$. Non-perturbative dynamics of $SU(2)$ group is of importance in this case. Fundamental representation of this group coincides with its conjugate, so that in low energy theory there are four fields (three $SU(3)$ -components of the field Q and \bar{R}) in this representation of $SU(2)$ group. This is the simplest example of SQCD with $N_F = N_C$. Deformation of the moduli space implies according to Eq. (37) that the origin of the field space cannot be the vacuum and the criterion of supersymmetry breaking is satisfied again. This limiting case of the “3-2” model illustrates dynamical supersymmetry breaking due to the deformation of the moduli space.

$\Lambda_2 \sim \Lambda_3$. In this case dynamics of both groups plays a role. The most general effective superpotential compatible with all symmetries, has the form,

$$W_f = \frac{\Lambda_3^7}{Y} + kX_1 + \mathcal{A}(Z - \Lambda_2^4). \quad (45)$$

The first term in this equation is the effective superpotential, generated, as discussed above, due to non-perturbative $SU(3)$ dynamics. The second term is the tree-level superpotential (42). In the third term, a new gauge invariant combination of fields appears,

$$Z = Q^3 \bar{R}.$$

A question arises, why this invariant had not been taken into account earlier, in the study of classical flat directions? It is sufficient to write down explicitly all group index contractions in the expression for Z in order to understand the answer,

$$Z = \epsilon^{abc} \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} Q_{\alpha a} Q_{\beta b} Q_{\gamma c} \bar{R}_{\delta}.$$

Greek letters denote $SU(2)$ indices here and Latin letters denote $SU(3)$ indices, ϵ is completely antisymmetric tensor. Making use of the identity for the product of ϵ -symbols,

$$\epsilon^{\alpha\beta} \epsilon^{\gamma\delta} = \delta^{\alpha\gamma} \delta^{\beta\delta} - \delta^{\alpha\delta} \delta^{\beta\gamma},$$

one obtains that the invariant Z is identically zero at the classical level. At the quantum level, vev's of the fields satisfy the constraint,

$$Z = \Lambda_2^4,$$

which is similar to Eq. (37). It is this constraint that leads to the deformation of the moduli space in the case $\Lambda_2 \gg \Lambda_3$. The last term in Eq. (45) is nothing but expression of this constraint in terms of a Lagrange multiplier. It is straightforward to see from Eq. (45) that the origin never belongs to the quantum space of vacua in “3-2” model and, consequently, supersymmetry is always broken here.

Let us now discuss one of the possible mechanisms of generating non-zero $\langle s \rangle$ and $\langle F_s \rangle$ in this model.

The theory exhibits global $U(1)$ symmetry with the following charges of the matter fields $Q(1/3)$, $\bar{L}_1(2/3)$, $\bar{L}_2(-4/3)$, $\bar{R}(-1)$. If this symmetry is gauged it can play a role of $U(1)_m$. It is worth noting, however, that this symmetry is anomalous in the original theory, so that it is necessary to include additional fields (e.g., field E with charge equal to $+2$) which are singlets under non-abelian groups. The key point of the mechanism of Ref. [32] is to use D -term of this group for generating non-zero $\langle s \rangle$ and $\langle F_s \rangle$.

Let us consider the case $\Lambda_2 \ll \Lambda_3$ for definiteness. Then the effective superpotential takes the form

$$W = kX_1 + \frac{\Lambda_3^7}{Y}.$$

All dimensionful parameters in this theory are characterized by the unique scale $v \sim \Lambda_3/k^{1/7}$. Namely, masses of the scalar fields are of order $m_s \sim kv$, and masses of vector fields are $m_v \sim gv$. Some of these fields are charged under $U(1)_m$ (e.g., the scalar component of X_2). One can demonstrate [32] that supersymmetry breaking results in non-zero vev of the D -term of this group,

$$\langle D \rangle = \frac{\alpha_m}{4\pi} m_s^2 \log \frac{m_v^2}{m_s^2}.$$

Then one can add a pair of fields Z_{\pm} with charges ± 1 under $U(1)_m$ and with the following superpotential,

$$W = \lambda_1 Z_+ Z_- S + \frac{\lambda_2}{2} E Z_-^2 + \frac{\lambda_3}{3} S^3. \quad (46)$$

As a result, rather cumbersome potential in the messenger sector arises,

$$V = \left| \frac{\partial W}{\partial \Phi} \right|^2 + 2\pi\alpha_m (\langle D \rangle + 2|e|^2 + |z_+|^2 - |z_-|^2)^2 - m_{z_{\pm}}^2 (4|e|^2 + |z_+|^2 + |z_-|^2),$$

where the dominant contributions to parameters $m_{z_{\pm}}, m_e^2$ at small $\lambda_1 \ll \alpha_m$ are given by two loop corrections due to abelian gauge interaction,

$$m_{z_{\pm}}^2 = -\frac{2\alpha_m^2}{\pi^2} m_s^2 \log \frac{m_v^2}{m_s^2}, \quad m_e^2 = 4m_{z_{\pm}}^2.$$

There is a region of parameter space where minimization of this potential results in non-zero vev's of the fields z_+, z_- and e , leading in turn to non-zero $\langle s \rangle$ and $\langle F_s \rangle$. The basic difficulty of this model is fine tuning of parameters, $\lambda_1 \ll \alpha_m$. This difficulty can be avoided in more complicated models [33].

Phenomenology of the models with $U(1)_m$ group is typical for gauge mediation models and was discussed in sections 3, 4. It is worth noting that supersymmetry breaking scale in the secluded sector F_{DSB} is rather high there, because it is related to Λ through small coupling constant $\alpha_m \lesssim 1$ of $U(1)_m$ group,

$$\Lambda^2 \sim \left(\frac{\alpha_m}{4\pi} \right)^2 F_{DSB}.$$

The corresponding gravitino mass at $\Lambda \sim 100$ TeV is

$$m_{3/2} = \frac{F_{DSB}}{\sqrt{3}M_{Pl}^*} \simeq 3 \text{ eV} \frac{F_{DSB}}{(100 \text{ TeV})^2} \gtrsim 0.5 \text{ keV}. \quad (47)$$

5.2.2 Secluded sector with vector-like matter

Recent rapid development of methods of investigation of supersymmetric theories made it possible to demonstrate that supersymmetry breaking may happen in models with only vector-like matter representations as well.

The simplest theory of this type [98, 97] is based on the $SU(2)$ gauge group with four fundamental fields Φ_i and six singlets which compose antisymmetric tensor Z^{ij} under $SU(4)_F$ group of global symmetries. Classical flat directions are parametrized by vev's of singlets Z^{ij} and by usual meson invariants $M_{ij} = \epsilon^{ab} \Phi_i^a \Phi_j^b$ subject to a classical constraint

$$\epsilon^{ijkl} M_{ij} M_{kl} = 0.$$

Let us add $SU(4)_F$ -symmetric superpotential

$$W_{tree} = \frac{1}{2} k \Phi_i \Phi_j Z^{ij}. \quad (48)$$

Fields Φ_i become heavy in the limit $kZ^{ij} \gg \Lambda_2$, where Λ_2 is the dynamical scale of the gauge group, and the low energy theory is described by supersymmetric $SU(2)$ gluodynamics with the effective dynamical scale

$$\Lambda_{eff}^6 = k^2 \Lambda_2^4 Z_{ij} Z^{ij}.$$

Gluino condensation takes place in this model (see section 5.1). As a result, the effective superpotential (31) is generated. The relevant effective scale $\Lambda_G = \Lambda_{eff}$ is the function of fields Z , so that O’Raifeartaigh type potential arises which breaks supersymmetry. At small values of Z and at $k = 0$ quantum deformation of moduli space takes place so that the origin does not belong to the quantum moduli space. This fact combined with the requirement $M_{ij} = 0$ following from stationarity of superpotential at $k \neq 0$ leads to supersymmetry breaking in this limit as well.

It was shown in Ref. [99] that it is possible to obtain non-zero $\langle s \rangle$ and $\langle F_s \rangle$ without additional $U(1)_m$ gauge interaction, but by making use of corrections to Kahler potential.

Let us rewrite the fields in terms of singlets (\hat{M} , Z) and five-plets of the flavour group $SP(4)_F \subset SU(4)_F$. Effective potential can be rewritten in the following form [99]

$$W_{eff} = \mathcal{A}(\hat{M}^2 + \hat{M}^a \hat{M}^a - \Lambda_2^4) + k_z Z \hat{M} + k Z^a \hat{M}^a ,$$

where \mathcal{A} is a Lagrange multiplier and constants k and k_z can be different if global symmetry $SP(4)_F$ is broken by some mechanism. We will consider a simple case when breaking of this symmetry does not affect supersymmetry breaking and leads to small values of k_z . Let us consider the following interaction of the secluded sector field with singlet,

$$W_{int} = \lambda_1 S \hat{M} - \frac{\lambda_3}{3} S^3 .$$

Fields Z^a and \hat{M}^a are massive with the mass $k\Lambda$ and can be integrated out. The resulting low energy superpotential is

$$W_{low} = k_z \Lambda_2^2 Z + \lambda_1 \Lambda_2^2 S - \frac{\lambda_3}{3} S^3 .$$

Kahler potential invariant under R-symmetry and discrete Z_2 -symmetry (\hat{M} , S , Z , λ_3 (as an external field) are odd under this symmetry) has the form,

$$K = ZZ^+ + SS^+ - \frac{\eta}{\Lambda_2^2} |k_z Z + \lambda_1 S|^4 - \lambda_1^2 \lambda_3^2 \frac{\xi}{\Lambda_2^2} |k_z Z + \lambda_1 S|^2 SS^+ + \dots ,$$

where positive constant η is of order one and loop factor $\xi \sim 10^{-3}$. It is the latter one which is responsible for mediation of supersymmetry breaking in the observable sector at non-zero λ_3 . Phenomenological parameters Λ and x (see Eq. (11)) are related to the parameters of the secluded sector as follows,

$$\Lambda \simeq \frac{\xi}{2} k_z^4 \lambda_1^{3/2} \lambda_3^{3/2} \Lambda_2 , \quad x \simeq \frac{\xi}{2} k_z^4 \lambda_1 .$$

It was shown that with the proper choice of parameters one can satisfy the limits on gravitino masses, and provide the stability of the Higgs potential.

5.2.3 Composite singlets

Rather cumbersome intermediate sector and additional $U(1)_m$ gauge group in the models considered above are related to the necessity to generate non-zero vev's $\langle s \rangle$ and $\langle F_s \rangle$ for singlet which interacts with messengers through superpotential $SQ\bar{Q}$. In several cases it may be simpler to deal not with the fundamental singlet S but with composite field [100] (interactions of the messengers with fundamental fields become non-renormalizable in this case). Phenomenology of the specific models can be rather different then because the secluded sector can affect low energy theory not only through non-zero $\langle s \rangle$ and $\langle F_s \rangle$.

5.2.4 Vacuum metastability

There are a number of theoretical problems which are common in gauge mediation models. These problems require new mechanisms to resolve them. One such problem is that the global minimum of the potential of the full theory including secluded and observable sector is actually supersymmetric and breaks MSSM gauge group in the majority of the models. Supersymmetry breaking minimum turns out to be metastable in this case.

For instance, it was shown in Refs. [101, 102] that in Dine-Nelson models considered in section 5.2.1, the global minimum of the potential is supersymmetric and is located at the point

$$\langle F_s \rangle = \langle s \rangle = 0, \quad \lambda' \langle e_+ e_- \rangle = -\lambda \langle \bar{q} q \rangle$$

when singlet-messenger interaction is taken into account. Generally speaking, this implies breaking of $SU(3) \times U(1)$ symmetry and phenomenologically acceptable vacuum discussed in section 5.2.1 is metastable in this case. However, its lifetime is longer then the age of the Universe at rather natural choice of parameters in these models.

Another way to solve this problem is to construct a model with the global supersymmetry breaking and colour conserving minimum. The simplest solution is to introduce explicit mass terms for messengers. The corresponding mass should be in the range $10^5 - 10^{15}$ GeV. Some new mechanisms are necessary then to provide the naturalness of this new scale. Alternatively, one can make use of additional singlets or new particles in the observable sector. However, the latter solution is incompatible with perturbativity of MSSM up to the Grand Unification scale [101].

Additional chiral fields can be used when there is an abelian gauge group rotating the fields Z_{\pm} (see section 5.2.1) [101]. However, a new problem arises in this case due to generation of the Fayet-Iliopoulos terms in the potential of the secluded sector. These terms lead to so-called mass instability due to mixing of $U(1)_m$ group with the MSSM hypercharge group [103].

It is worth to discuss this problem in more detail. Mixing of the kinetic terms is possible for the abelian gauge groups. In supersymmetric case it leads to mixing

of corresponding D-terms. The mixing term $F_1^{\mu\nu} F_2^{\mu\nu}$ is renormalizable and can be present in the effective Lagrangian without being suppressed by a dimensionful factor. Mixing of the D-terms implies the generation of unacceptably large (of the order of supersymmetry breaking scale in the secluded sector) scalar masses for MSSM particles. Some additional symmetry can forbid this mixing. Alternatively, one can use more complicated gauge group for fields Z_\pm instead of $U(1)_m$, so that mixing would be impossible.

A successful attempt to obtain a theory with global supersymmetry breaking vacuum due to variation of the secluded sector has been presented in Ref. [104]. $SU(2)$ vector model of section 5.2.2 was used to break supersymmetry, and $U(1)_m$ was one of the abelian subgroups of $SP(4)_F$ global symmetry group. Fields Z_\pm acquire soft positive masses due to threshold effects of $U(1)_m$ gauge interaction. Non-zero $\langle s \rangle$ and $\langle F_s \rangle$ appear due to the Higgs mechanism after Yukawa interaction of singlet with Z_\pm is taken into account. In spite of the presence of additional abelian group, the problem with mixing of D-terms does not arise in this case because this mixing is forbidden by the charge conjugation symmetry of the original theory.

5.3 Models with direct mediation

The approach described in section 5.2 allows one to construct phenomenologically viable models with gauge mediation. Some shortcomings of this approach, in particular, those discussed at the end of the previous section, call for modification of these models. One of the approaches is to allow messengers to carry quantum numbers not only of the Standard Model but also of the secluded sector gauge group. Such theories are called models with direct mediation. In this case messengers themselves participate in the dynamics of supersymmetry breaking (hence another name – models with dynamical messengers).

This approach certainly looks more natural and historically the first attempts to construct realistic models with dynamical supersymmetry breaking [82] exploited this scheme. There are some problems, however, which lead to additional complications in model building, when the mechanism of direct mediation is involved. The main difficulty is the following. Let messengers transform as some representation of the secluded sector gauge group. This group plays a role of a flavour group from the point of view of the visible sector, i.e., several copies of messenger fields appear in the spectrum. The number of copies coincides with the dimension of the corresponding representation of the secluded sector gauge group. Since messengers carry Standard Model quantum numbers, they give contribution to the β functions of gauge coupling constants of QCD and electroweak theory, that may result in the loss of asymptotic freedom of QCD. If the dimension of the representation of the secluded sector gauge group, i.e., the number of messenger copies, is larger than four in the case of $SU(5)$ GUT representation $5 + \bar{5}$ (or one for $10 + \bar{10}$) and messenger threshold $M \lesssim 10^8$ GeV, then the coupling constants of the Standard Model groups become

large below 10^{16} GeV. This fact contradicts the attractive idea of their perturbative unification. The majority of models with dynamical supersymmetry breaking known up to now do not lead to representations of dimension less than four¹³, that had even provoked some authors to call this way “not the most clever” [32]. Nevertheless, models with direct mediation of supersymmetry breaking are studied intensively and several realistic and rather elegant theories have been found already.

The approaches to solve the problem with asymptotic freedom of the Standard Model in theories with dynamical messengers are conventionally separated into three types. The most developed approach (see section 5.3.1) deals with heavy messengers: if the messenger scale is sufficiently high, then Landau pole occurs above the GUT scale and perturbative unification of coupling constants remains intact. Two other approaches look less conventional. One of them is based on the refusal to describe physics up to GUT scale in terms of the Standard Model. If observed matter fields represent composite low energy degrees of freedom of some effective theory whose coupling constant becomes strong at the intermediate scale between M_W and M_{GUT} , then the fundamental matter content might differ from the content at low energies. The observed particles may be composed of very small number of fundamental constituents. The Standard Model gauge group would be asymptotically free at high energies in this case despite of a large number of charged degrees of freedom in the visible low energy spectrum. This possibility has not been studied in great detail, we will discuss this scenario in section 5.3.2. Finally, the last approach is to refuse to unify MSSM coupling constants in perturbative domain in favour of the unification in the strong coupling regime. Such a possibility does not contradict phenomenological requirements and the idea of Grand Unification; moreover, this way is even more preferable from several points of view. The general features of this scenario have been discussed in section 4.3.2.

5.3.1 Heavy messengers

We turn now to the discussion of the models with direct mediation, where the perturbative unification of the Standard Model gauge coupling constants is achieved due to the large values of thresholds pushing Landau pole beyond the GUT scale. First, note that the use of heavy messengers is consistent with sufficiently large soft masses of superpartners of the Standard Model particles. The reason is that it is the *ratio* $\Lambda = \langle F_s \rangle / \langle s \rangle$ that determines soft masses, see Eqs. (12), (13), while the messenger mass scale is determined by $\langle s \rangle$. Messengers do not affect running of the Standard Model coupling constants below the scale $\langle s \rangle$. This value may be large at phenomenologically viable $\Lambda \sim (100 \text{ TeV}) / \sqrt{n}$, where n is the effective number of messengers. As discussed in section 3.3, the main constraints on the messenger mass come from cosmology and give $\langle s \rangle \lesssim 10^{11}$ GeV.

¹³The recently proposed model [105] of direct mediation with only two messenger copies is discussed in section 14.

How to make messengers heavy? The standard method to get large masses of messengers is to provide large vacuum expectation value for the scalar component of the superfield S , which interacts with messengers Q, \bar{Q} through Yukawa superpotential $SQ\bar{Q}$. The general scheme is the following. Let some flat direction be parametrized by the field ¹⁴ S . If the superpotential is generated along this direction due to quantum effects (as in case of SQCD at $N_f < N_c$ considered in section 5.1.2), then one has run-away vacuum, i.e., the energy reaches its minimum only at $S = \infty$. To obtain finite vacuum expectation value of S , one needs a competitive contribution to the potential, which would lift the flat direction at large S . The corresponding terms are either included in the superpotential “by hands”, or generated due to quantum effects.

Let us consider a supersymmetric flat direction, along which the scalar potential is zero. The potential for singlet S may appear due to nonperturbative effects in the following way. Let S interact, apart from Q, \bar{Q} , with fields f, \bar{f} from the secluded sector,

$$W_f = S f \bar{f},$$

where n flavours (\bar{f}) f transform as the (anti)fundamental representation of some asymptotically free gauge group $SU(m)$, whose gauge coupling becomes strong at the scale Λ . The fields f, \bar{f} effectively become heavy at the values of scalar component $S \gg \Lambda$ and have to be integrated out from the low energy theory. If the dynamics of gauge group $SU(m)$ dominates and there are no other fields charged under this group, then the low energy theory is a supersymmetric Yang-Mills theory where gaugino condensation provides the nonperturbative superpotential

$$W_{\text{eff}} = \Lambda_{\text{eff}}^3,$$

with the scale of effective theory Λ_{eff} determined from the matching condition

$$\Lambda_{\text{eff}}^{3m} = S^n \Lambda^{3m-n}. \quad (49)$$

The powers of Λ on the right and left hand sides of this formula are the first coefficients of the β function below and above the matter field thresholds, respectively; in this case S plays a role of mass of heavy fields and the power of S is determined by dimensional arguments. Thus, the effective superpotential depends on S and is equal to

$$W_{\text{eff}} = \Lambda^{3-n/m} S^{n/m}.$$

The perturbative expansion for the Kahler potential of S about the canonical expression $K_S = S S^\dagger$ is valid far from $S = 0$. The corresponding contribution to the scalar

¹⁴More exactly, by a holomorphic gauge invariant composed of the components of this field: although S is the Standard Model singlet, it may transform nontrivially under the secluded gauge group in direct mediation models.

potential has the form

$$V_{\text{eff}} = |F_s|^2 = \left| \frac{\partial W}{\partial s} \right|^2 = \Lambda^{(6-2n/m)} S^{2(n/m-1)}, \quad (50)$$

so the potential is independent of S at $n = m$, i.e., the flat direction is uniformly lifted forming a plateau with constant scalar potential. At $n < m$ the corresponding potential would push the vacuum away to large S . In the latter case, a contribution of the different origin is required to stabilize the flat direction and to obtain a vacuum with nonzero but finite $\langle s \rangle$ and $\langle F_s \rangle$. The simplest way is to add the terms increasing with S directly into the tree-level superpotential. Such terms usually have to be nonrenormalizable in order that the nonsupersymmetric minimum appears at large S . Nonrenormalizable operators in the tree superpotential have to be suppressed by a dimensional parameter whose physical meaning is an energy cutoff, where this description fails. One usually implies that this parameter is the Planck mass, therefore the contribution of nonrenormalizable operators is small. This leads to the required minimum of the scalar potential at very large S , because the stabilizing effect is weak in comparison with “repulsion” to the region $S \rightarrow \infty$. Large S automatically provides large values of messenger masses. The other way to stabilize the flat direction is to involve the dynamics of one more gauge group, if it generates the increasing potential. Similar arguments are valid in more complicated models, in particular, when the field S is not a singlet.

We turn now to the case $n = m$ in Eq. (49), when nonperturbative effects provide positive and independent of S scalar potential (50), i.e., there is a flat direction of inequivalent supersymmetry breaking minima (recall this statement is valid only at large S , since near $S = 0$ the corrections to Kahler potential are uncalculable and may lead to additional contributions to the scalar potential. In particular, these contributions may restore supersymmetry, therefore the minima with nonzero vacua might be local). We have to take into account the next order contributions, namely, perturbative corrections to the Kahler potential, because the potential does not depend on S along flat direction in the leading order. Had we studied a supersymmetric flat direction, these corrections would not alter the scalar potential along it: they are multiplied by $F_S = 0$ in the scalar potential. In our case, $F_S = \Lambda^2 \neq 0$, therefore taking into account wave function renormalization gives (small) corrections to the potential, which may provide isolated vacuum. This mechanism to obtain vacuum at large values of S is known as *inverse hierarchy* [106]. As will be shown below, to exploit this mechanism one needs a nonsinglet field S transforming as a nontrivial representation of an asymptotically free group.

If the field S transforms under the gauge group G , then a nonzero vacuum expectation value of the scalar component of this field breaks the group down to subgroup G' . In the one-loop approximation, the Kahler potential for S gets contributions due to the loops generated by fields Q , \bar{Q} , f , \bar{f} and heavy vector fields, whose mass is of order S when G is broken down to G' (the gauge fields of G' do not contribute

to the Kahler potential at the one loop level, since they do not interact with S). If g is the gauge coupling constant of G , and λ is the Yukawa constant of interaction $SQ\bar{Q} + Sf\bar{f}$, then in one loop approximation

$$K(S, S^\dagger) = SS^\dagger \left(1 + (c_g g^2 - c_\lambda \lambda^2) \log(SS^\dagger/\Lambda_G^2) \right), \quad (51)$$

where $c_{g,\lambda}$ are positive coefficients and Λ_G is a cutoff parameter. The leading logarithms may be summed at large S by making use of the renormalization group, then constants g , λ will be changed to their effective values at the scale S . With this contribution taken into account the potential takes the form

$$V_{\text{eff}} = \frac{\left| \frac{\partial W}{\partial S} \right|^2}{\frac{\partial^2 K}{\partial S \partial S^\dagger}}. \quad (52)$$

The behaviour of the potential is determined by running of the coupling constants. The Yukawa constant often grows at large energies providing the growth of scalar potential at infinity; the gauge coupling of the asymptotically free group G' grows at low energies. The combination of the two effects provides a maximum of the coefficient in front of SS^\dagger in (51) at a value of S which corresponds to the minimum of the effective potential (52). Because of the slow variation of logarithm, this minimum corresponds to large S , that is to heavy messengers, as in the mechanism with nonrenormalizable terms.

We have described above several ways to obtain the vacuum with large $\langle s \rangle$ and nonzero $\langle F_s \rangle$. These approaches were exploited to construct concrete models, which incorporate dynamical supersymmetry breaking and its direct mediation to MSSM through gauge interactions.

Models with nonrenormalizable terms in the superpotential. To make use of the direct mediation mechanism, one has to satisfy certain requirements on the supersymmetry breaking sector. In fact, as it has already been mentioned, a given set of matter fields and their interactions should have a global flavour symmetry group large enough to include the Standard Model group; the latter has to remain anomaly free after being gauged. Finally, the group has to remain unbroken despite of supersymmetry breaking. To know whether this is the case, the exact information about the supersymmetry breaking vacuum is required.

The theories with the gauge group $SU(N) \times SU(M)$, whose infrared dynamics was discussed in Refs. [107], are exploited as a supersymmetry breaking sector in some of these models [108, 109]. It is difficult to obtain an exact information about infrared dynamics at $M < N - 2$. The global minimum of the potential is supersymmetric at $M = N$ and these models are incorporated in schemes where the supersymmetry breaking vacuum is local (see below). The theories with $M = N - 1$ and $M = N - 2$ break supersymmetry and their low energy dynamics is under control.

Unfortunately, these theories seem to be phenomenologically unacceptable due to the following reasons. As has been pointed out in Ref. [108] and discussed in detail in Ref. [110], masses of scalar superpartners of the Standard Model particles are not determined by the formulae from section 3 in this case. The formulae (12), (13) were derived under the assumption of quite specific supersymmetry breaking messenger spectrum: masses of components of supermultiplet are splitted in such a way that $\text{STr } m_{\text{mess}}^2 = 0$. This condition is not always valid in models with direct mediation and the main contribution to the masses of scalar particles of MSSM is given by the logarithmically divergent diagrams, which are proportional to $\text{STr } m_{\text{mess}}^2$ and the logarithm of an ultraviolet cutoff,

$$\delta m^2 \sim -g^4 (\text{STr } m_{\text{mess}}^2) \log \frac{\Lambda_M}{m}. \quad (53)$$

Here Λ_M is the scale of ultraviolet cutoff, m is the mass of light messengers. One can see that the MSSM soft terms depend significantly on the high energy physics; Eq. (53) gives larger contribution than Eq. (13). Since $\text{STr } m_{\text{mess}}^2 > 0$ in these models, large negative contribution to the masses squared of sleptons and squarks appears. This contribution exceeds positive one coming from the usual diagrams (Fig.1) and leads to breaking of the gauge group of the Standard Model, the fact that makes these theories phenomenologically unacceptable.

To avoid breaking of the Standard Model group due to the contribution (53), it is necessary to ensure that there is no light particles with large soft masses charged under this group. One may follow one of two ways. First, one can exclude all light charged fields from the secluded sector, for instance, by lifting all flat directions parametrized by fields with nontrivial quantum numbers of the Standard Model at the level of nonrenormalizable operators (see, e.g., Ref. [111]). Second, one can provide large soft masses for scalar components of these light fields, so that the positive $\text{STr } m_{\text{mess}}^2$ does not appear; this is the case in models with local vacuum.

Models with local vacuum. It has already been pointed out that the basic mechanism providing a local vacuum at large fields is the inverse hierarchy. The general scheme of this mechanism have been formulated in Ref [40] and its essence is the following¹⁵. The gauge group consists of three factors $G_S \times G_B \times G_w$. Roughly speaking, the strong group G_S provides supersymmetry breaking, the weak G_w contains the Standard Model group, and asymptotically free “balancing” group G_B is required for the inverse hierarchy. The field S is charged only under G_B and there is one flat direction, parametrized by an invariant composed of the components of S . G_B is broken to some subgroup H_B along this flat direction. The fields Q, \bar{Q} become heavy at large S and all arguments presented in the beginning of this section are correct.

¹⁵We consider only models with direct mediation, though this scheme can be applied with slight variations to the models where supersymmetry breaking is mediated via singlet [40].

The supersymmetry breaking vacuum in all these models is false, so a natural question arises, whether this vacuum is stable for cosmological intervals of time? We have already concluded in section 3 that such a possibility is not excluded, i.e., lifetime of a metastable vacuum can be much larger than the age of the Universe. This lifetime Γ is given by the semiclassical exponent in these models [40],

$$\Gamma \propto e^{-S_B}, \quad S_B \sim 2\pi^2 \left(\frac{\langle s \rangle}{\Lambda} \right)^2.$$

One obtains that the lifetime of supersymmetry breaking metastable state exceeds the age of the Universe at $\langle s \rangle/\Lambda \gtrsim 10$.

Gauge messengers. Unfortunately, there are specific contributions to the masses squared of the MSSM scalars appear in specific models with the inverse hierarchy mechanism (see e.g., Ref. [112, 40]). These contributions are related to the fact that $G_B \times G_w$ is broken at the scale $M_{\text{mess}} \ll M_{GUT}$ to a subgroup which includes the Standard Model group. The masses squared of squarks and sleptons become negative due to these terms [113], so that colour and electromagnetic groups are broken. Hence, these models are not realistic.

The presence of other fields (besides ordinary messengers) that mediate supersymmetry breaking to the visible sector is responsible for this phenomenon. These fields in adjoint representation of the Standard Model group are known as gauge messengers. These are heavy vector fields, which obtain masses as a result of breaking $G_B \times G_w \rightarrow (SU(3) \times SU(2) \times U(1))_{SM}$ by the vacuum expectation value of $\langle s \rangle$.

The method proposed in Ref. [113] allows to trace an appearance of the negative contribution to the masses squared. Namely, the effective number of messengers n , entering the expressions for soft masses, equals the value of a jump of the first coefficient of β function at the threshold which corresponds to messenger masses — in case of fundamental messengers it is merely the number of additional flavours N_m . If the symmetry breaking $G \rightarrow H$ occurs at the same scale, then

$$n = N_m - 2(C_G - C_H), \quad (54)$$

where C_G and C_H are contributions to the first coefficient of β function for the adjoint representations of G and H , respectively. The origin of the coefficient (-2) in Eq. (54) is the contribution from heavy vector bosons (-3) and would-be Goldstone bosons $(+1)$. Thus, the contribution of gauge messengers is negative even at one loop level, and may lead to $n < 0$. This statement is valid also in the case of breaking $G_B \times H \rightarrow H'$ (where all matter fields of the Standard Model are assumed to be charged under H , H' groups).

Models without gauge messengers. There are various ways to overcome this difficulty. First, one can consider the same model but with additional matter charged

under the balancing group G_B (or one can suppose that a part of the Standard Model matter is charged under H' , while another part is charged under G_B). Then C_G becomes smaller and dangerous negative contribution to masses squared of superpartners proportional to \sqrt{n} decreases. Unfortunately, “weakening” of the balancing group may destabilize the minimum obtained by making use of the inverse hierarchy, or may shift this minimum to the unacceptable region of small S . Phenomenologically viable models with gauge messengers are still unknown.

The other method is to modify the models in such a way that gauge messengers are absent, that is symmetry breaking $G_B \times H' \rightarrow G_{SM}$ does not occur at the scale M_{mess} . In other words, the field S providing the supersymmetry breaking masses to messengers Q, \bar{Q} is neutral under G_B . At first sight, this idea contradicts the inverse hierarchy, because the latter requires a charged field parameterizing a lifted flat direction. The elegant solution was proposed in Ref.[114]. Let the field S be a singlet. Let, however, the interaction in superpotential relates its vacuum expectation value $\langle s \rangle$ to the vacuum expectation value of another operator, which in turn breaks G_B . The flat direction, which the inverse hierarchy mechanism works along, is parametrized by a *linear combination* of the singlet interacting with messengers and another field charged under the balancing group. Massive vector bosons appearing as a result of breaking of G_B are neutral under the Standard Model group, so they do not serve as gauge messengers.

Since the balancing group is separated from the Standard Model, it is possible to simplify the gauge sector of the theory. However, additional degrees of freedom and new parameters are required, which describe interaction of fields which enter the linear combination mentioned above. An example of a model based on this trick is a theory with $SU(5)$ strong group and balancing group $SU(2)$ with one fundamental flavour $\psi, \bar{\psi}$ (any asymptotically free theory with vector matter may serve as a balancing group) [114]. Five messenger generations Q, \bar{Q} contribute to the masses of superpartners of the Standard Model particles. To make this fact consistent with perturbativity of coupling constants, messengers have to be heavy that may contradict Eq. (19).

To obtain correct hierarchy of the parameters in the Higgs sector of MSSM, one might include two additional singlets with a specific superpotential. The price for this solution of the μ -problem is the introduction of new parameters without fine tuning their values.

A completely different solution, associated with a significant simplification of a model, has been proposed recently [105] and deals with identifying G_B and G_S . Namely, diagonal $SU(2)_D$ subgroup of strong $SU(2) \times SU(2)$ group plays the role of G_B . There are some fields ($\Sigma(2, 2)$ and six flavours of $Q(2, 1)$ and $\bar{Q}(1, 2)$) charged under the strong group and coupled through the superpotential $\Sigma Q \bar{Q}$. The inverse hierarchy mechanism works along the flat direction parametrized by $\det \Sigma$. The Standard Model gauge group is embedded into $SU(6)$ global symmetry of the secluded sector, so that this model does not contain gauge messengers but only two generations of the ordinary messengers in the fundamental representation of $SU(5)$ GUT group. Thus,

the problem with loss of asymptotic freedom of the Standard Model gauge group does not arise; however, the numerical estimates demonstrate [105] that viable region of parameters of this model corresponds to heavy messengers anyway, $M_{\text{mess}} \gtrsim 10^{10}$ GeV (otherwise Landau pole for Yukawa coupling $\Sigma Q\bar{Q}$ would emerge at too low energies). Nevertheless, a range of suitable parameters, 10^{10} GeV $\lesssim M_{\text{mess}} \lesssim 10^{11}$ GeV, survives. Note that additional mechanism is required in this model to solve the μ -problem.

Thus we have seen that, apparently, only a small number of theories with heavy messengers does not lead to breaking of the Standard Model gauge symmetry and may be considered as realistic. Unfortunately, these viable models have too heavy messengers, the fact which conflicts with constraints from nucleosynthesis (19). Note that these constraints may be reconsidered (see above and Ref.[40, 41]); moreover, there is an example of a model [105] where a small region of cosmologically acceptable messenger masses remains.

5.3.2 Composite models

The idea that quarks and leptons are low energy degrees of freedom of an effective theory that is strongly coupled at a certain energy scale (similar to π -mesons, which are low energy degrees of freedom of QCD), is rather popular because it allows both to solve aesthetic problems of the Standard Model and to expect new physics at energies significantly lower than the GUT scale. Since dynamical supersymmetry breaking also exploits strong dynamics beyond the Standard Model, it might be natural to expect that one and the same strongly coupled gauge group is responsible for the compositeness as well as for supersymmetry breaking.

The way to understand that compositeness may help to solve the problem of perturbative unification of coupling constants even in models with a large number of messengers is the following. The matter fields of MSSM and (or) messengers are assumed to be low energy composite degrees of freedom of the strongly coupled secluded sector. Some fundamental degrees of freedom (preons) are charged under the Standard Model gauge group, moreover, preons belong to the complete multiplets of GUT group (e.g., $SU(5)$). Let the fundamental gauge theory be strongly coupled at the scale Λ_S . Then at higher energies the contributions to the β functions of the Standard Model gauge coupling constants come only from preonic degrees of freedom, while at lower energies only matter fields of the visible sector give such a contribution. There is a threshold at the energy Λ_S where one set of degrees of freedom replaces the other. It is possible (combinatorically) to construct a number of low energy degrees of freedom from a few independent preons, therefore the low energy theory may be asymptotically non-free (many messengers). Coefficients of β functions may change, however, yet in the weak coupling domain of the MSSM; at high energies the theory is asymptotically free and gauge couplings remain small in this case. As both fundamental and composite fields belong to complete $SU(5)$ multiplets, the perturbative unification of the coupling constants remains intact (at least, at the one

loop level) – the first coefficients of β functions of all three Standard Model groups would be shifted by the same number.

To construct composite models with perturbative unification of coupling constants, one has to be able to analyze the low energy description of strongly coupled gauge theory in order to learn which composite degrees of freedom enter the effective theory, to describe the vacuum and to confirm that the Standard Model group is not broken in this vacuum. For our purposes, this vacuum has to break supersymmetry and the fields mediating supersymmetry breaking to MSSM fields have to appear. The methods of studying the supersymmetric gauge theories described in section 5.1 allow one to carry out this analysis in some cases (see, e.g., Ref. [115]). In models with composite quarks and leptons originated from the supersymmetry breaking sector, superpartners of matter fields get soft masses due to the dynamics responsible for supersymmetry breaking; global symmetries provide universality of soft terms. Gaugino masses may arise from the loop diagrams due to the interaction either with messengers or with quarks and leptons. Messengers are not required at all in the latter case, but gaugino might be too light.

To complete this section, it is worth pointing out that models with direct mediation have been invented to simplify theory in comparison with models with mediation via singlet. However, specific direct mediation models are something even more contrived.

6 Gauge mediation and cosmology

Let us consider cosmological implications of gauge mediation theories. In particular, we discuss the following topics:

1. Gravitino as a candidate for dark matter.
2. Moduli and dilaton in the early Universe.
3. The role of new particles (messengers and fields from the secluded sector) in the evolution of the Universe.

6.1 Low energy supersymmetry breaking and light particles

As it has been mentioned above, the relatively low scale of supersymmetry breaking in the secluded sector $F_{DSB} \lesssim (10^{10} \text{ GeV})^2$ is typical for models with gauge mediation. This leads to light gravitino and, in compactified string theories, to light masses for some other fields.

6.1.1 Light gravitino

Lower bounds on $m_{3/2}$. Various lower bounds on light gravitino mass are set by low energy measurements and astrophysics. The strongest bound, however, comes

from physics at accelerators. Let us recall the conservative limit $\Lambda \gtrsim 3 \times 10^4 \frac{1}{\sqrt{n}}$ GeV, presented in section 3.3 (n is the effective number of messenger generations). This limit implies the estimate

$$m_{3/2} = \frac{F_{DSB}}{\sqrt{3}M_{Pl}^*} > \frac{\Lambda^2}{\sqrt{3}M_{Pl}^*} = 0.5 \frac{1}{n} \text{eV}.$$

The lower bound on gravitino mass may be significantly higher in specific models [116]. For example, this is the case, if the singlet S gets its vacuum expectation value through interaction with $U(1)_m$ charged fields from the secluded sector (the mechanism discussed in section 5.2.1). If the gauge coupling constant α_m is assumed to be small up to M_{GUT} then

$$\frac{\alpha_m}{4\pi}(\Lambda) \lesssim \frac{1}{b_m \log \frac{M_{GUT}}{\Lambda}},$$

where b_m is a sum of $U(1)_m$ charges squared of messenger fields (usually this value is of order 10). Hence we have $\alpha_m \sim 0.2$, and

$$m_{3/2} \gtrsim 70 \text{keV} \frac{1}{nk_m^2} \frac{0.2}{\alpha_m} \left(\frac{m_{\tilde{e}_R}}{45 \text{GeV}} \right)^2,$$

where k_m is a coefficient in the relation $\Lambda = k_m \frac{\alpha_m}{4\pi} \sqrt{F_{DSB}}$. For $n = 1 \div 4$ we obtain $m_{3/2} \gtrsim 100 \text{ keV}$, $\Lambda \simeq 3 \times 10^4 \text{ GeV}$.

Constraints on $m_{3/2}$ related to the evolution of the Universe. The contemporary picture of the evolution of the Universe is most sensitive to the particle physics when primordial nucleosynthesis and energy density of the Universe are discussed.

NLSP decoupled from the visible sector before the beginning of nucleosynthesis (i.e., before 1 s after Big Bang). Photons from their decay, however, might affect significantly the abundance of chemical elements in the Universe because the light nuclei would be destroyed. One may avoid this problem if the reheating temperature after inflation was not too high, so that dangerous concentration of superpartners was not reached. Another solution is based on the requirement that NLSP had to decay during the first second of the evolution of the Universe. In the latter case one obtains the following constraint on the parameters of the theory [24],

$$\sqrt{F_{DSB}} \lesssim 10^5 \left(\frac{m_{NLSP}}{100 \text{ GeV}} \right)^{5/4} \text{ TeV},$$

thus the models with heavy gravitino ($m_{3/2} \gtrsim 5 \text{ GeV}$, that is $F_{DSB} \sim m_{3/2} M_{Pl}^* \gtrsim (10^6 \text{ TeV})^2$) look the least promising from the cosmological point of view.

Another limit on the gravitino mass comes from the requirement that the average energy density of the Universe, $\Omega \rho_c$, does not exceed the critical value ρ_c . For instance, assuming that gravitino is thermalized in the early Universe, one obtains the bound

$m_{3/2} < 2h^2 \text{keV}$ [117], where h is Hubble constant expressed in units 100km/s/ Mpc . At $h \sim 0.7$ only gravitino lighter than 1 keV is acceptable. In this case we obtain an upper bound on the scale of supersymmetry breaking: $\sqrt{F_{DSB}} < 2 \times 10^3 \text{ TeV}$ [60]. Together with the lower bound coming from physics at accelerators (see section 4.2), we find that gravitino would not be cosmologically dangerous in the model with NLSP decaying inside a detector.

It is known from observation of rotation curves of galaxies that the significant part of an average density of the Universe consists of invisible matter. In addition, consideration of nucleosynthesis implies that the main part of the dark matter is non-baryonic. *Light gravitino in thermal equilibrium may play the role of warm dark matter* [117].

The typical gravitino masses are $1 \text{ keV} \lesssim m_{3/2} \lesssim 100 \text{ keV}$ in Dine-Nelson type models. In this case NLSP would decay relatively late and gravitino might overclose the Universe. This does not occur in the cosmological models with late inflation, where at the reheating temperature $T_r < \tilde{m}_{NLSP} \sim m_Z$, multiple production of NLSP and increasing of number of gravitino [58] do not occur. We note, that *since gravitino from NLSP decays are not thermalized, it might be a candidate for cold dark matter* [118].

In the models with direct mediation (see section 5.3), gravitino heavier than 100 keV are typical, and the main contribution to their production rate comes from scattering $A + B \rightarrow C + \psi_{3/2}$.

The compilation of bounds on the reheating temperature are shown on Fig. 3 (see Ref. [116]). We see that $T_{max} \ll 10^8 \text{ GeV}$, which is hardly possible in the usual inflation scenario. Therefore, at $m_{3/2} \gtrsim 1 \text{ keV}$ some additional entropy production is required to reduce the gravitino density. One of the mechanisms of entropy production deals with the relaxation of the fields corresponding to the flat directions of MSSM potential. If the energy accumulated in the field parameterizing the flat direction dominates the energy of the Universe, then the decay of the field increases the temperature of the Universe up to $1 \div 10 \text{ TeV}$. In this case gravitino are not thermalized and required entropy production may occur via fine tuning of parameters responsible for the decay; in addition, the baryon-photon ratio remains at the acceptable level.

In the general case, the large entropy production may wash out the baryon number. This is not the case if baryogenesis is provided by means of the Affleck-Dine mechanism [119, 120]¹⁶.

Example of a model. The model proposed in Ref. [123] and based on the $SU(2)$ -model (see section 5.2.2) in the secluded sector is one of the most suitable from the cosmological point of view. Recall that in the low energy theory, the only singlet Z remains which acquires nonzero vacuum expectation value of the auxiliary component

¹⁶The electroweak baryogenesis [121], taking place also at relatively low temperature, is not suitable in this case because of large $m_{\tilde{t}}$ [122].

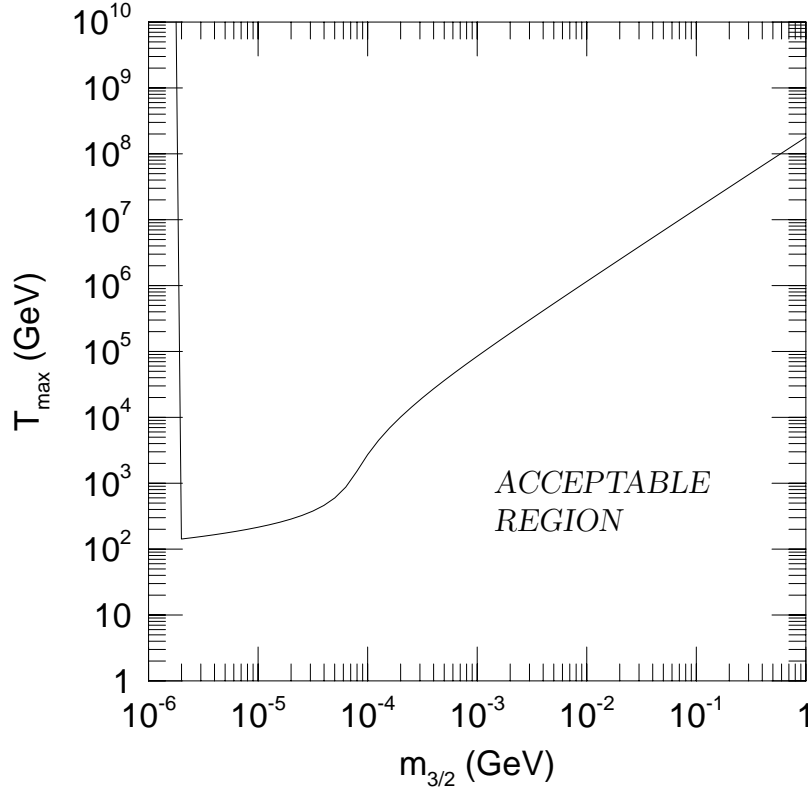


Figure 3: The upper bounds on the reheating temperature T_{max} from the requirement $\Omega_{3/2} h^2 < 1$ at various gravitino masses. [116]

due to the superpotential $W = k_z \Lambda_{DSB}^2 Z$ ($\Lambda_{DSB} = \Lambda_2$ is a scale of $SU(2)$ group) and acquires nonzero $\langle z \rangle$ due to the Kahler potential.

One takes the superpotential as in the classical work by O’Raifeartaigh [11]:

$$W_1 = Z (k_z \Lambda_{DSB}^2 + \lambda Q \bar{Q}) + m Q \bar{Q}' + m' Q' \bar{Q} \quad (55)$$

where (Q, \bar{Q}) , (Q', \bar{Q}') are two vector messenger generations. There is a non-supersymmetric vacuum with $\langle F_z \rangle = k_z \Lambda_{DSB}^2$ and $k_z \langle z \rangle \sim \Lambda_{DSB}$. This vacuum is stable provided $|mm'| > |\lambda \langle F_z \rangle|$.

Masses of gauginos and scalars will be given again by formulae similar to Eqs. (12) and (13), where $\Lambda = \frac{\lambda \langle F_z \rangle}{\sqrt{mm'}}$, and $x = \frac{\lambda \langle F_z \rangle}{mm'}$. However, an additional suppression appears due to a function with a maximal value 0.1 in the prefactor. At small x , the gaugino masses turn out to be suppressed by additional factor x^2 due to an accidental cancellation. As a result, the strongest limits on the parameters of the theory are

taken from searches for gaugino [48]: $M_2 \gtrsim 50$ GeV, $M_3 \gtrsim 220$ GeV. Taking into account the upper bounds on squark masses we obtain for the parameter x that $1 \geq x^2 \gtrsim 0.1$, and for the gravitino mass, $m_{3/2} = \frac{\langle F_z \rangle}{\sqrt{3}M_{Pl}^*} \sim \frac{1}{\lambda} x^{-5} 10^{-2}$ keV, one has

$$10 \text{ eV} \lesssim m_{3/2} \lesssim 3 \text{ keV}$$

at $\lambda \sim 1$ and $\Lambda_{DSB} \simeq 10^{5 \div 6}$ GeV.

In all arguments above we assume $m \sim m' \sim \Lambda_{DSB}$. Certainly, this relation may occur to be correct in the strong coupling regime. One can avoid explicit introduction of mass terms by including additional fields in the theory. Thus, the model discussed above is an example of the theory where gravitino may constitute warm dark matter in the framework of the standard scenario of inflation.

6.1.2 Cosmology in the context of string theories

Moduli. It is often assumed that supersymmetric gauge theories are spontaneously compactified string theories, an assumption which may lead to specific cosmological problems. A part of them is related to the presence of so-called moduli fields in string theories. The expectation values of moduli fields parametrize the compactified dimensions in the coordinate space. The flat directions of the potential correspond to moduli fields in the theory with unbroken supersymmetry. Supersymmetry breaking generates moduli mass $m_\Phi \sim m_{3/2}$. The evolution of the moduli is important for the models with hidden sector [124] (so-called Polonyi problem) as well as for the models with low energy supersymmetry breaking [125]. We consider only models of the latter type; the general analysis of this problem was performed in Ref.[126].

The essence of the moduli problem is the following. Since their potential is almost flat, moduli fields evolve slowly from their initial values, $\Phi \sim M_{Pl}$, which may result in overclosing the Universe. Indeed, the critical density decreases during the expansion of the Universe. Moduli interact weakly with other fields; at the early stage their energy density receives the main contribution from the nonzero potential and remains almost constant. In this case the energy density plays a role of vacuum energy (cosmological constant) and does not lead to slowing down the expansion of the Universe. However, at Hubble constant $H \sim m_\Phi$ moduli fields begin to oscillate and to produce particles, whose density may exceed the critical value at this moment.

One of the possible solution to this problem [116, 128] is an additional late inflationary stage (for instance, a stage of the thermal inflation [129]). Moduli fields are diluted at this stage, and baryon asymmetry decreases. However, the Affleck-Dine mechanism of baryogenesis [119] may lead to a significant baryonic asymmetry [116] in this case. In order that this mechanism works successfully, the relic moduli density has not to be less than

$$\Omega_\Phi h^2 \gtrsim 10^{-6} \frac{m_\Phi}{100 \text{ keV}} \quad (56)$$

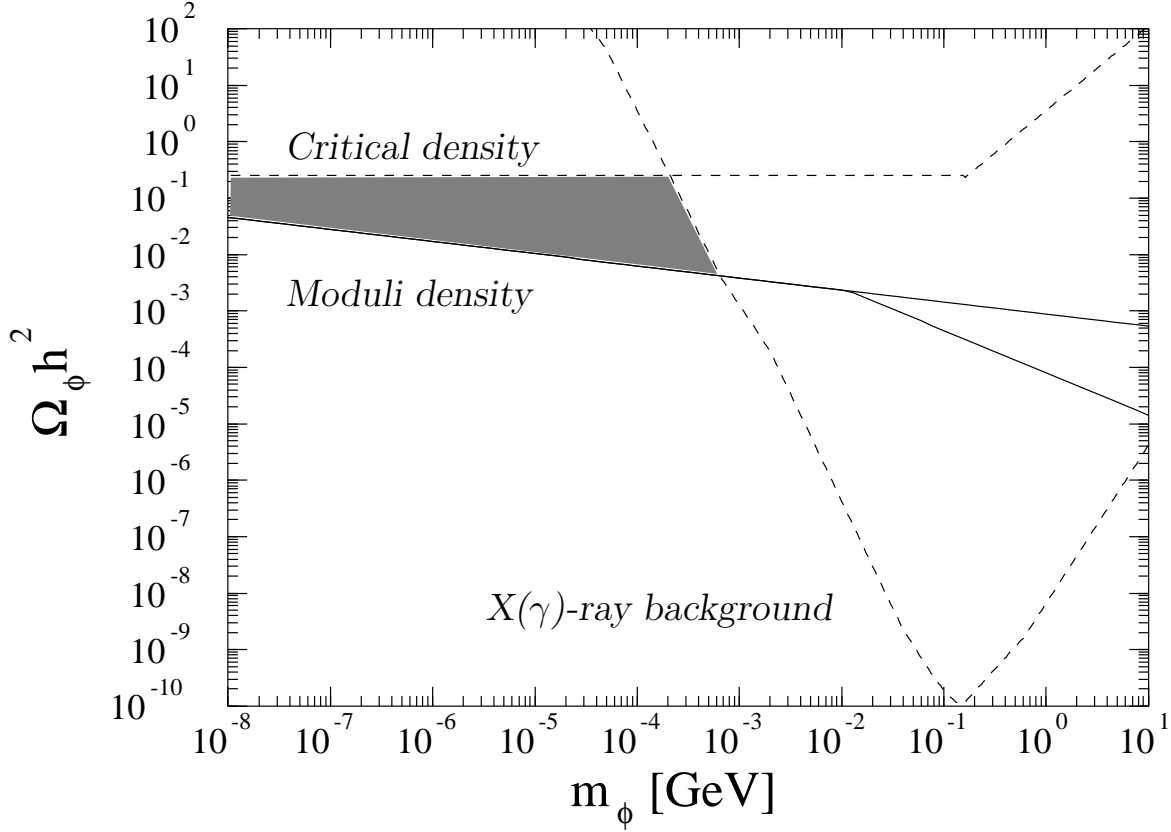


Figure 4: The limits on the moduli mass [127] in model with modified thermal inflation; acceptable region is painted in grey.

In specific models (see, e.g., Ref. [127]) stronger lower bound on the moduli density may arise due to their additional production after thermal inflation epoch.

The additional constraint comes from measuring the flux of cosmic rays and is related to the fact that the main channel of moduli decay is pair production of γ -quanta. Combining this limit with bound (56) one obtains an upper bound on the moduli mass, $m_\phi \lesssim 2$ MeV [130].

The set of limits on the parameters of moduli is presented in Fig. 4 (see Ref. [127]). Although these results are obtained in a specific model (modified thermal inflation) many of them are valid in other cases.

It was found that solution of all these problems without introduction of additional energy scales is possible only in the framework of rather complicated inflationary models (see, e.g., Ref. [127]). In this case, the Affleck-Dine mechanism provides sufficient baryon production ($\frac{n_B}{s} \gtrsim 10^{-10}$) [116].

It is worth noting that the hypothesis of low energy supersymmetry breaking leads, as a rule, to new long-range forces originating from the interaction via light

moduli fields [59, 131]. This prediction may be checked in experiments.

Dilaton. Similar problems arise for the dilaton field ϕ . One expects that this field acquires mass of order $m_{3/2}$ due to nonperturbative dynamics [126]. Dilaton lifetime is much longer than the age of the Universe. One can use [116] the mechanism of late thermal inflation [129] in order to reduce the energy accumulated by the dilaton field below the critical value. It was found that for the inflaton mass exceeding 130 GeV the lower bound on the ratio of dilaton energy density to entropy is reached at the minimal reheating temperature $T_r = 10$ MeV, which is acceptable for successful nucleosynthesis. The value of present critical density $\frac{\rho_c}{s} = 3.6 \cdot 10^{-9} h^2$ GeV leads to a limit [132]

$$\frac{\rho_\phi}{\rho_c} h^2 \equiv \Omega_\phi h^2 \gtrsim 1.5 \cdot 10^{-2} \left(\frac{m_\phi}{\text{MeV}} \right)^{-3/4}.$$

The dilaton energy density can be less than critical at $m_\phi \gtrsim 20$ keV in this case. If the inflaton mass is not larger than 130 GeV, then [132]

$$\Omega_\phi h^2 \gtrsim 2.3 \cdot 10^{-3} \left(\frac{m_\phi}{\text{MeV}} \right)^{-3/14}.$$

It means that in the region $m_\phi \simeq 10$ MeV-1 GeV the lower bound on $\Omega_\phi h^2$ is lower than current critical density $\Omega h^2 \simeq 0.25$ by a factor of about 30.

For both dilaton and moduli, the strongest constraints come from the analysis of the observed γ -ray flux, which dilaton decays $\phi \rightarrow 2\gamma$ contribute to. In the case of the simplest interaction Lagrangian

$$\mathcal{L} \sim \frac{1}{16\pi\alpha} \frac{\Phi}{M_{Pl}^*} F_{\mu\nu}^2$$

the dilaton lifetime is $\tau_\phi \simeq 7 \cdot 10^{23} \left(\frac{\text{MeV}}{m_\phi} \right)^3$ s, and the dilaton masses from 500 keV to 1 GeV contradict the experiment [130]. It is natural to expect $m_\phi \sim m_{3/2}$ that provides a strong upper bound on the value of supersymmetry breaking F-term: $\sqrt{F_{DSB}} < 5 \cdot 10^7$ GeV. A possibility to reconcile string compactification with rather wide class of models under discussion is excluded by this limit.

6.2 Stable particles and cosmology

Let us consider the cosmological consequences of the presence of new fields (messengers and fields from the secluded sector) in the model.

First, we consider the secluded sector. The conservation of possible global charges in the secluded sector guarantees the stability of the lightest charged particle, in analogy to the baryon number in the Standard Model which ensures proton stability. Following Ref. [35] we consider the constraints on the parameters of the theory, assuming that new stable particles (“baryons”) form dark matter. One can estimate

the density of the relic particles from their total annihilation cross section to light neutral particles [133],

$$\Omega_B \cdot h^2 \gtrsim \left(\frac{m_B}{300 \text{ TeV}} \right)^2.$$

Hence, “*baryons*” from the secluded sector with $m_B \sim 100$ TeV are realistic candidates for cold dark matter, the latter playing key role in the contemporary picture of the structure formation in the Universe.

We turn now to the study of messenger fields. In the case of indirect mediation and in the absence of mixing discussed in section 4.4 these fields are separated from the secluded sector by singlet S and from the visible sector by gauge interactions. It is not difficult to see that in the case of indirect interaction and in the absence of mixing discussed in section 4.4, a global quantum number carried by messengers is conserved. The lightest of these fields will be also stable and may contribute to dark matter. If the lightest stable messenger were electrically charged, its presence would result in the distortion of the spectrum of cosmic rays [134]. To find the lightest messenger, we consider mass splitting between electrically charged and neutral components of messengers in the fundamental representation. This splitting arises at the tree level due to the contribution of $SU(2)$ D -term, as a result of electroweak symmetry breaking,

$$(m_+^2 - m_0^2)_{tree} = \sqrt{(\lambda \langle F_s \rangle)^2 + \frac{1}{4} M_Z^4 \cos^2 2\beta} - \sqrt{(\lambda \langle F_s \rangle)^2 + \left(\sin^2 \theta_w - \frac{1}{2} \right)^2 M_Z^4 \cos^2 2\beta}. \quad (57)$$

This splitting is of order $M_Z^2 \frac{M_Z^2}{\lambda \langle F_s \rangle}$. The one loop contribution to the masses is proportional to αM_Z^2 . Which contribution dominates, is determined by the ratio of suppression factors α and $\frac{M_Z^2}{\lambda \langle F_s \rangle}$. Assuming that messengers are heavier than electroweak bosons, i.e. $\frac{\Lambda^2}{x^2} (1-x) \gtrsim 1 \text{ TeV}^2$, the one loop contribution to mass splitting between electrically charged and neutral components of messenger fields takes the form [35]:

$$(m_+^2 - m_0^2)_{1-loop} = \frac{\alpha}{4\pi} M_Z^2 \left(4 \log \frac{x}{1-x} - \log \frac{1+x}{1-x} + \frac{2x}{1-x} \log \frac{2x}{1+x} - 4 \right). \quad (58)$$

From the analysis of Eqs. (57) and (58) one infers whether the lightest messenger is neutral.

The condition $m_+ > m_0$ does not guarantee the absence of problems with charged particles. Decay width of the charged messenger component q^+

$$\Gamma(q^+ \rightarrow q^0 e \nu) = \frac{G_F^2}{15\pi^3} (m_+ - m_0)^5$$

is strongly suppressed by a small factor $(\delta m)^5$. Late decay of a charged particle may affect nucleosynthesis and result in discrepancy with observed matter abundances in

the Universe. Nucleosynthesis is not spoiled if the lifetime of q^+ is of order 1 s. From this requirement one obtains

$$(m_+ - m_0) \gtrsim 5\text{MeV} \quad (59)$$

One more constraint comes from the condition that the density of the stable messengers should not exceed the critical value. From the requirement $\Omega h^2 < 1$, one obtains the limit on the mass of the lightest messenger,

$$m_0^2 \lesssim 25 \text{ TeV}^2. \quad (60)$$

By comparing the inequalities (59), (60) with Eqs. (57) and (58), we find the small region in the entire parameter space (Λ, x) where the lightest stable messengers may be phenomenologically acceptable, namely, $x \gtrsim 0.95$. The dominant contribution here is Eq. (58).

Introducing several messenger generations with small weak mixing and different parameters (Λ_i, x_i) may extend somewhat the range of parameters. In this case a similar constraint should be imposed on $\max\{\Lambda_i\}$ and does not affect the parameters of the lightest messenger with $\Lambda_{\text{light}} \ll \max\{\Lambda_i\}$, which is cosmologically acceptable under one of the following conditions: $\sqrt{\langle F_s \rangle_{\text{light}}} \lesssim 350 \text{ GeV}$ (in this case (57) and (59) are compatible; (57) dominates provided x is not very small; this region is phenomenologically unacceptable in models with one singlet S) or $x_{\text{light}} \gtrsim 0.95$.

The extension to the case of messengers belonging to the other representations is straightforward. However, it does not result in extending the allowed parameter region.

The possibility that messengers constitute dark matter may be tested in direct searches for dark matter. The lightest messenger scatters off nuclei through Z -boson exchange. Searches for dark matter imply that such particles with masses 5 TeV compose not more than a quarter of mass of the Galactic halo with density 0.3 GeV/cm^3 . It means, that *it is impossible to solve the dark matter problem by making use of messenger fields only, even invoking fine tuning.*

To conclude, the stable lightest messenger is unacceptable for cosmology for most values of parameters. In analogy to gravity mediation models this problem may be solved by invoking late stage of inflation and low reheating temperature (of order of M_Z). The violation of global messenger quantum numbers due to mixing with MSSM fields considered in section 4.4 seems to be more natural.

7 Conclusions

We considered various models which invoke a beautiful mechanism of gauge mediation of supersymmetry breaking. We discussed phenomenology of these models and their cosmological consequences. The basic distinctive features of this approach are the following.

- In models of gauge mediation, it is possible to describe mediation of supersymmetry breaking to the observable sector and to calculate parameters of this breaking just in the frameworks of field theory. It is not necessary to investigate the effects of quantum gravity or string theory. Models with gauge mediated supersymmetry breaking have very few free parameters; theoretical predictions of these models are not in conflict with experimental data.
- A necessary ingredient of these models is a set of messenger fields, that is relatively heavy ($\gtrsim 10$ TeV) matter fields which transform as vector-like representations of the Standard Model gauge group.
- Supersymmetry breaking occurs at relatively low energies ($\lesssim 10^{10}$ GeV) and mass spectrum of superpartners of the Standard Model fields is determined by their quantum numbers and values of gauge coupling constants. As a consequence, (a) flavour changing processes are naturally suppressed without fine tuning; (b) the lightest superpartner is gravitino, this fact results in specific phenomenology.
- To obtain a correct pattern of the electroweak symmetry breaking, fine tuning or introduction of additional parameters is required these models (μ -problem).

Several specific realizations of the gauge mediation mechanism were constructed. The difference among them concerns the ways to include the supersymmetry breaking sector. We point out the following characteristic features.

- The “Minimal” model does not include additional parameters in the Higgs sector but requires fine tuning of couplings responsible for electroweak symmetry breaking. The way out is to introduce additional parameters.
- Specific realistic models where supersymmetry breaking sector is incorporated often require complicated dynamics. In a series of models, supersymmetry is broken in a metastable vacuum.
- Relatively simple models with direct mediation require a large number of messengers. Many models with heavy messengers are unrealistic because they break $SU(3) \times U(1)$ and are problematic for cosmology. The large number of light messengers results in lost of perturbativity of coupling constants at the scale of Grand Unification.

Therefore, many theories of this kind are consistent with plethora of experimental limits. However, in our opinion, there is no model which satisfies constraints coming from both physics at accelerators and cosmology and which does not require fine tuning. This feature is undesirable since the natural origin of parameters is one of the cornerstones of the models with gauge mediation. We would like to list a few interesting problems in theories with gauge mediated supersymmetry breaking, problems which probably will be solved in the nearest future:

- It would be interesting to find new ways to constrain the parameters in the frameworks of each specific model. As a rule, this requires to consider both the visible and the secluded sectors of the theory. An example of such a constraint is provided by boundary conditions at the high energy scale according to the Grand unification or string compactification (see e. g. [135, 71]).
- Relatively strong constraints on gauge mediation models come from cosmology, so it would be important to make the considered mechanism consistent with models of the early Universe without fine tuning.
- Many models with gauge mediated supersymmetry breaking are not studied well enough from the phenomenological point of view. Detailed analysis may lead to unexpected consequences and make several models phenomenologically inconsistent.
- Challenging approaches to the standard problems of model building, such as refusal of the perturbative unification of gauge coupling constants, may simplify the theories without loss of viability.

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Note added. The Russian version of this review has been submitted to *Uspekhi* in 1998. Since then, a number of papers appeared where phenomenological [136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147], model building [148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158], and cosmological [159, 160, 161, 162, 163] aspects of gauge mediated supersymmetry breaking are discussed.

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